0366.3267 Graph Theory

Fall Semester 2022

Homework assignment 4

Due date: Sunday January 22, 2023 (**SHARP!**)

Problem 1. Prove: $R(3,4) = 9$.

Problem 2. Let m be given. Show that if n is large enough then every n-by-n $0,1$ matrix has a principal submatrix of size m in which all elements above the diagonal are the same, and all elements below diagonal are the same.

Problem 3. Prove that for every positive integer k there is an integer n such that in every k-coloring of all subsets of $[n]$ there are disjoint non-empty subsets A, B such that the sets $A, B, A \cup B$ have the same color.

Problem 4. Prove that for every tree T and integers $k \geqslant 2$, $g \geqslant 3$, there exists a graph G without cycles of length up to g and such that every k-coloring of the edges of G contains a monochromatic copy of T.

Problem 5. Let G be a graph with n vertices, m edges and t triangles. Show that $3t \geqslant 4m^2/n - mn$. Derive: $ex(n, K_3) \leqslant n^2/4$.

Problem 6. 30 people need to place a call to each other using their cellular phones (one call per each pair). A cellular phone company gets 1 shekel for each call between two people at distance between 800 and 1000 meters. The company is allowed to locate the people as it wishes so as to maximize its profit. What is the maximum possible profit of the company?