

GRAPH THEORY

Michael Krivelevich

Fall Semester 2024

Course number: 0366.3267.

When and where: Sundays 15-18, Kaplun 118.

Prospective audience: the course is intended for second and third year undergraduate students in Mathematics or Computer Science.

Prerequisites: first year courses in mathematics, most notably Discrete Mathematics or Introduction to Combinatorics.

Requirements and grade: Homeworks will be given once every two-three weeks; submitting at least three quarters of them is mandatory; the final grade will be composed from the final exam's grade (90%) and homeworks (10%).

Syllabus

1. Basic notions: graph, graph isomorphism, adjacency and incidence matrices, paths, cycles, connectivity, subgraphs. Vertex degrees, Sperner's lemma and Brouwer's fixed point theorem.
2. Trees. Equivalent definitions of a tree. Spanning trees. Cayley's formula, matrix-tree theorem.
3. Connectivity. Vertex connectivity and edge connectivity. Degrees and connectivity, Mader's theorem. Blocks, 2-connected graphs. Menger's theorem.
4. Eulerian and Hamiltonian graphs. Theorems of Dirac, Ore, Chvátal-Erdős.
5. Matchings. Theorems of Hall and König for bipartite graphs. Tutte's theorem.
6. Colorings. Vertex colorings, degrees and chromatic number. Greedy algorithm. Brooks' theorem. Color-critical graphs. Graphs with high girth and high chromatic number. Edge coloring, theorems of König and Vizing.
7. Ramsey theory. Ramsey's theorem. Upper and lower bounds for Ramsey numbers. Examples of Ramsey-type theorems.
8. Extremal graph theory. Turán's theorem. Kővari-Sós-Turán theorem.
9. Planar graphs. Euler's formula. Coloring planar graphs. Kuratowski's theorem.

Bibliography

1. B. Bollobás, Modern Graph Theory, Springer, 1998.
2. J. A. Bondy and U. S. R. Murty, Graph theory, Springer, 2008.
3. R. Diestel, Graph theory, Springer, 2024, or earlier editions.
4. L. Lovász, Combinatorial problems and exercises, 2nd edition, North-Holland, 1993.
5. D. West, Introduction to graph theory, 2nd edition, Prentice Hall 2001.