ADVANCED COMBINATORICS – POSITIONAL GAMES

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Course number: 0366.4005
When and where: Sun. 16-19, location: Shenkar (physics) 222.
Prospective audience: the course is intended for graduate and advanced undergraduate students in Mathematics or Computer Science.
Prerequisites: Discrete Mathematics, Introduction to Probability. Familiarity with basic concepts of Graph Theory will be very helpful.

COURSE DESCRIPTION

I guess all of you played the game of Tic-Tac-Toe (Crosses and Noughts on the 3-by-3 board) in your early childhood, and probably some of you spent your time playing Tic-Tac-Toe on a larger board (and probably none of you tried its high-dimensional version...). Here is another very popular game – the game of Hex, played on a rhombus of hexagons of size $n \times n$, where the two players, White and Black, take the two opposite sides of the board each, and then take alternatively unoccipied hexagons of the board, trying each to connect his opposite sides of the board; whoever achieves this first wins. Another example: the game is played on the edges of the complete graph K_n on n vertices, Player I (Maker) takes each time one unoccupied edge, Player II (Breaker) responds by taking q unoccupied edges. Maker wins if he creates of copy of a fixed graph G from his edges, otherwise the win is of Breaker.

All the above games and amazing variety of other games can be cast in the following uniform framework. Let p, q be positive integers, and let $H = (E_1, \ldots, E_k)$ be a collection of finite set (a hypergraph). The board of the game is the set of vertices of H. Two players take turns occupying previously untaken vertices of H. Player I takes in his turn p unoccupied vertices, Player II responds by taking q unoccupied vertices. The edges of H are winning sets. The game is specified completely by defining who wins in every final position. For example, whoever occupies completely an edge of H wins; or the first player wins if he takes an edge of H by the end of the game, while the second player aims to prevent him from achieving his goal; or the first player loses if he occupies an edge of H in the end and wins otherwise. Those are **Positional Games**.

Positional Games stand at the interface of Game Theory and Combinatorics. Their study involves a variety of combinatorial arguments of different sorts, including sometimes probabilistic reasoning and intuition. This subject has been promoted and popularized mainly by József Beck, whose papers will serve us as the main bibliographic sourse in this course.

List of (recent) papers of J. Beck on positional games

1. J. Beck, Achievement games and the probabilistic method, *Combinatorics*, Paul Erdős is Eighty, Keszthely, Vol. 1, Bolyai Soc. Math. Studies, (1993), 51–78.

2. J. Beck, Deterministic graph games and a probabilistic intuition, *Combinatorics, Probability and Computing* 3 (1994), 13–26.

3. J. Beck, Foundations of positional games, i*Random Structures and Algorithms*, 9 (1996), 15–47.

4. J. Beck, Games, Randomness and Algorithms, in: **The Mathematics of Paul Erdős**, R. L. Graham and J. Nesetril, eds., Springer, Berlin, 1996, 280–310.

5. J. Beck, Ramsey games, *Discrete Math.* 249 (2002), 3–30.

6. J. Beck, Positional games and the second moment method, *Combinatorica* 22 (2002), 169–216.

7. J. Beck, The Erdős-Selfridge theorem in positional game theory, Bolyai Soc. Math. Studies, 11: Paul Erdős and His Mathematics. II Budapest, 2002, 33-77.

8. J. Beck, Tic-Tac-Toe, Contemporary Mathematics, Bolyai Soc. Math. Studies 10 (2002), 93–137.