ADVANCED SEMINAR IN COMBINATORICS – PSEUDO-RANDOM GRAPHS

Michael Krivelevich

Spring Semester, Academic Year 2006-7

Seminar number: 0366.4538
When and where: Mon. 15-17, location: Orenstein 102.
Prospective audience: the seminar is intended for graduate and advanced undergraduate students in Mathematics or Computer Science.

SEMINAR DESCRIPTION

Random graphs is one of the most central concepts of modern Combinatorics, studied extensively for its own sake and also due to its amazing applicability to a wide range of topics in Combinatorics and Computer Science. The most studied model is undoubtedly the so called binomial random graph $G(n, p)$; this is the probability space of all labeled graphs on $1, \ldots, n$, where each pair of vertices $i \neq j$ is an edge independently and with probability $p = p(n)$.

The utmost importance of random graphs, combined with their simple yet somewhat elusive definition based on pure randomness, suggest naturally to try and capture most essential properties of random graphs through quantitative, usually deterministic requirements. This leads to the notion of **pseudo-random graphs** (sometimes also called quasi-random graphs). Informally speaking, a pseudo-random graph $G = (V, E)$ is a graph on n vertices, whose properties resemble closely those of a truly random graph $G(n, p)$ with the same edge density $p(n) = |E(G)|/\binom{n}{2}$. Of course, it matters much which exactly properties of random graphs we would like to imitate, and how exactly we measure and assure the proximity of pseudo-random graphs to their truly random counterparts.

Between the specific topics we plan to discuss in this seminar are:

- Basic properties of random graphs;

- The notion of jumbled graphs due to Thomason;

- Equivalent definitions of weak pseudo-randomness;

- Approach through eigenvalues; the notion of (n, d, λ)-graphs;

- Constructions of pseudo-random graphs;

- Properties of pseudo-random graphs.

The seminar will be largely based on a survey paper on the subject by M. Krivelevich and B. Sudakov, available at:

http://www.math.tau.ac.il/∼krivelev/prsurvey.ps

and additional research papers.