Positional Games 0366-4991

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Homework 1
Due: Apr. 15, 2015

1. Analyze the following strong game: the board of the game is $X = \{1, \ldots, 9\}$, and the family of winning sets $\mathcal{F}$ consists of all triples of X summing to exactly 15.
Remark. This is mostly a mathematical joke.

2. Prove that in the row-column game played on the $d \times n$ board, First Player can reach advantage d, if n is large enough compared to d.

3. Prove that the strong connectivity game played on the edges of the complete graph K_n is First Player's win, in several different ways, such as: a direct proof, a proof using Lehman's theorem, a proof using strategy stealing, a proof based on the Erdős-Selfridge criterion.

4. Prove that for any k there are $\epsilon > 0$ and n_0 such that for all $n > n_0$ First Player has a strategy to win the strong clique game (K_n, K_k) in at most $\left(\frac{1}{2} - \epsilon\right)\binom{n}{2}$ rounds.

5. In the Maker-Breaker arithmetic progression games $AP(n, k)$ the board is $[n]$ and the winning sets are all k-term arithmetic progressions contained in $[n]$. Prove that there are constants $c_1, c_2 > 0$ such that if $k < c_1 \log_2 n$ then the game is Maker's win, and if $k > c_2 \log_2 n$, then the game is Breaker's win.

6. In the Maker-Breaker coloring game $c(n, k)$ the board is $E(K_n)$, and Maker wins if and only if by the end of the game the graph of his edges is not k-colorable. Prove that there are constants $c_1, c_2 > 0$ such that if $k < c_1 \frac{n}{\log_2 n}$, then Maker wins, and it $k > c_2 \frac{n}{\log_2 n}$, then Breaker wins.