Positional Games 0366-4991

Michael Krivelevich
Spring Semester 2015

Homework 2
Due: May 13, 2015

1. Show that there exists a constant $c > 0$ such that for every graph $G = (V, E)$ with $|V| = n$ and $\Delta(G) \leq c \ln n$, Breaker wins the $(1 : 2)$ Maker-Breaker connectivity game on $E(G)$.

2. Prove that every pair of positive integers d, k there exists n_0 such that for every $n > n_0$ the following is true. Let $(X, \mathcal{F})$ be a k-uniform hypergraph on $|X| = n$ vertices, in which every element $x \in X$ belongs to at least one and at most d edges $A \in \mathcal{F}$. Then Maker wins the $(2 : 1)$ Maker-Breaker game on $\mathcal{F}$.

3. Prove that for every integer $k \geq 2$ there are $\epsilon, n_0 > 0$ such that in the $(1 : b)$ Maker-Breaker game on $E(K_n)$, for $n > n_0$ and $b \leq n^{\epsilon}$, Maker has a strategy to create a copy of K_k.

4. Show that there exist constants $c, n_0 > 0$ such that for $n > n_0$ and $b \leq \frac{cn}{\ln n}$, Maker has a winning strategy in the $(1 : b)$ perfect matching game on $E(K_{n,n})$.

5. Consider the following $(1 : b)$ Maker-Breaker game on $E(K_n)$: Maker's goal is to create a graph containing as large a connected component as possible. Prove that for constant $\epsilon > 0$ and large enough n, if $b \leq (1 - \epsilon)n$, then Maker has a stragegy to create a connected component of size linear in n, and if $b \geq (1 + \epsilon)n$, then Breaker has a strategy to keep all connected components in Maker's graph of size at most C, for some $C = C(\epsilon)$.