Positional Games 0366-4991

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Homework 3
Due: May 27, 2015

1. Show that for every $\epsilon > 0$ there is n_0 such that the following is true. Let $G = (V, E)$ be a graph on $|V| = n > n_0$ vertices with minimum degree $\delta(G) \geq \left(\frac{1}{2} + \epsilon\right) n$. Then Maker wins the $(1 : b)$ Maker-Breaker connectivity game on $E(G)$ for $b \leq \frac{n}{2 \log_2 n}$.

2. Prove that for every $\epsilon > 0$ and integer $k > 0$ there is n_0 such that for every $n > n_0$, Maker has a strategy to create a spanning k-connected graph, playing the $(1 : b)$ Maker-Breaker game on $E(K_n)$ for $b \leq \frac{(1-\epsilon)n}{\ln n}$.

3. Let t be a positive integer. Argue that, for any $\epsilon > 0$ and all large enough n, in the $\left(1 : (1-\epsilon)\frac{n}{\ln n}\right)$ Maker-Breaker game on $E(K_n)$, Maker has a strategy to create t edge-disjoint Hamilton cycles.

4. Let P be the property "G has a triangle and a Hamilton cycle". Let b_P be the threshold bias of the $(1 : b)$ Maker-Breaker game on $E(K_n)$ where Maker wins if in the end his graph belongs to P. Prove: $b_P = \Theta(\sqrt{n})$.

5. For a positive integer n, define the *rake* R_n to be the graph on $2n$ vertices obtained by taking the path P_n on n vertices and attaching an edge to each vertex of the path, where the other endpoints of these edges are distinct and disjoint from P_n. Prove that there are $\delta, n_0 > 0$ such that for every $n > n_0$ Maker can create a copy of R_n playing against Breaker in the $\left(1 : \frac{\delta n}{\ln n}\right)$ game on $E(K_{2n})$.