Positional Games 0366-4991

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Homework 4

Due: July 1, 2015
(submit your solution electronically or by placing it in my mailbox)

1. [biased Lehman does not work for MB, CW games] Prove that for every positive integer t there exists a t-edge connected graph G such that Breaker wins the $(1:2)$ Maker-Breaker connectivity game on $E(G)$, and Waiter wins the $(1:1)$ Client-Waiter connectivity game on $E(G)$.
Hint. Box Game...

2. Show that for every positive integer r there exist $c>0$ and infinitely many n for which there is an r-uniform hypergraph $(X,\mathcal{F})$ on $|X|=n$ vertices with $f^+_{\mathcal{F}} - f^-_{\mathcal{F}} \geq cn$, where $f^-_{\mathcal{F}}$, $f^+_{\mathcal{F}}$ are the lower and the upper threshold biases, respectively, for the Avoider-Enforcer game on $\mathcal{F}$ under strict rules.

3. Prove that there is an absolute constant $c>0$ such that Avoider wins the $(1:q)$ Avoider-Enforcer triangle game on $E(K_n)$ for $q \geq cn^{3/2}$, under both strict and monotone rules.

4. Let $k=k(n)$ be the maximal size of a clique that Waiter can force Client to construct in the $(1:1)$ Waiter-Client game on $E(K_n)$. Show that there exist constants $c_1, c_2 > 0$ such that for all large enough n, $c_1 \ln n \leq k(n) \leq c_2 \ln n$.

5. Let T be a tree on $k \geq 2$ vertices. Prove that if $q \leq cn^{\frac{k}{k-1}}$ for some $c=c(k)>0$ and n is large enough, then Waiter can force Client to construct a copy of T in the $(1:q)$ Waiter-Client game on $E(K_n)$.
Hint. Prove by induction on k, proving a stronger (counting) statement.