Random Graphs 0366-4767

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Homework 1

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1. Let $\tilde{G} = (G_i)_{i=0}^{\binom{n}{2}}$ be a random graph process. Prove that **whp** in $\tilde{G}$ the first triangle appears before the graph becomes connected.

2. Let $\epsilon > 0$ be a constant. Prove that **whp** the random graph $G(n, m)$ with $m = (1 + \epsilon)n$ is not planar.

3. Let H be a graph with a cycle. Prove that there exists a constant $a = a(H) > 0$ such that $t(n, H) \geq n^{1+a}$ for all sufficiently large n, where $t(n, H)$ is the Turán number of H, which is the maximum number of edges in an H-free graph on n vertices.

4. Prove that **whp** in $G(n, 0.5)$,

$$\omega(G), \alpha(G) = (1 + o(1))2\log_2 n\ .$$

5. Prove that **whp** in $G(n, 0.5)$, $h(G) = O(n/\sqrt{\log n})$, where $h(G)$ is the Hadwiger number of G.

6. A *subdivision* of a graph H is obtained by replacing the edges of H by internally disjoint paths of positive lengths. Hajós conjectured (thus strengthening Hadwiger's conjecture) that if G is k-chromatic, then G contains a subdivision of the complete graph K_k. What can you say about Hajos' conjecture when applied to the random graph $G(n, 0.5)$?