Random Graphs 0366-4767

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Homework 2

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1. Prove that for all k, ℓ there exists a *regular* graph G such that G is not k-colorable, but has no cycles of length at most ℓ.
Hint. Consider the random regular graph $G \sim G_{n,r}$ for $r = r(k)$ large enough; bound from above $\alpha(G)$.
2. Prove that for a fixed $r \geq 3$, a random r-regular graph $G_{n,r}$ is **whp** connected.
Remark. Using similar techniques, one can prove that $G_{n,r}$ is in fact **whp** r-connected.
3. Prove that for $G \sim G(n, \frac{c}{n})$ with $0 < c < 1$, **whp** every connected component of G contains at most one cycle.
Hint. Prove first that a connected graph with more than one cycle contains two vertices connected by three internally disjoint paths, or two vertex disjoint cycles connected by a path, or two cycles sharing exactly one vertex.
4. Prove that for every $\epsilon > 0$ there exists $\delta > 0$ such that the random graph $G \sim G(n,p)$ with $p \geq \frac{1+\epsilon}{n}$ satisfies **whp**: if C is the largest connected component of G, then $|E(C)| - |V(C)| \geq \delta n$.
Hint. Sprinkle, sprinkle...