

Random Graphs 0366-4767

Michael Krivelevich
Fall Semester 2010

Homework 4
Due: Jan. 30, 2011

1. Prove that for any constant $c > 0$ and integer k , if $G \sim G(n, c/n)$, then **whp** $\chi_g(G) > k$.
Hint. Expose the edges of G in $k + 1$ phases, in each phase exposing the edges incident to linearly many vertices. Prove that **whp** after phase i color C_i has linearly many vertices.
2. (a) A graph G is called d -degenerate if every subgraph of G has a vertex of degree at most d . Prove that if G is a d -degenerate graph, then $\chi(G) \leq d + 1$.
(b) Prove that for every constant $c > 1$, if $G \sim G(n, c/n)$, then **whp** $\chi(G) \leq 20c$.
3. For a graph $G = (V, E)$ denote by G^2 the graph with vertex set V and edge set $F = \{(u, v) : \text{dist}_G(u, v) \leq 2\}$. Let $G \sim G(n, n^{-2/3})$. Prove that **whp** $\chi(G^2) \geq \frac{cn^{2/3}}{\log n}$, where $c > 0$ is an absolute constant.