

0366.4817 Graph and Hypergraph Coloring

Spring Semester 2022

Homework assignment 3

Due date: Monday, June 5, 2022

Remark. In questions concerning chordal graphs you can use the characterization of chordal graphs through simplicial elimination ordering stated in the class.

Problem 1. Prove: every interval graph is a chordal graph.

Problem 2. Given a permutation σ of $[n]$, the permutation graph $G = G(\sigma)$ is a graph with vertex set $[n]$, where $i < j$ are connected by an edge iff $\sigma(i) > \sigma(j)$. Prove that permutation graphs are perfect.

Problem 3. Let G be a graph on n vertices. Prove: $\text{ch}(G) + \text{ch}(\bar{G}) \leq n + 1$.

Problem 4. Let $G = (V, E)$ be a graph, and let $L = (L(v))_{v \in V}$ be a list assignment for $V(G)$, satisfying:

$$\sum_{v \in V} \frac{1}{|L(v)|} \leq 1.$$

Prove that G is L -choosable.

Problem 5. Show that every chordal graph G is $\chi(G)$ -choosable.

Problem 6. Let G be a graph with $\alpha(G) = 2$. Prove: $\text{ch}(G) = \chi(G)$.