GRAPH AND HYPERGRAPH COLORING

Michael Krivelevich

Spring Semester 2022

Course number: 0366.4817.
When and where: Mondays 16-19, Orenstein 110.
Prospective audience: the course is intended for graduate and advanced undergraduate students interested in Combinatorics.
Prerequisites: working knowledge of graph theory, as provided by standard graph theory courses; familiarity with basic notions of probability and linear algebra.
Requirements and grade: Homeworks will be given once every two-three weeks; final grade will be based on grades for homeworks.

Syllabus

(Subject to changes/detailization as the course progresses)

1. Basic notions and definitions: vertex and edge coloring, chromatic number and chromatic index; coloring infinite graphs, de Brujin-Erdős theorem; basic bounds on the chromatic number, Nordhaus-Gaddum theorem; Gallai-Hasse-Roy-Vitaver theorem.
2. Degrees and coloring: degeneracy, greedy coloring, Szekeres-Wilf theorem; Brooks' theorem.
3. Equitable coloring, Hajnal-Szemerédi theorem.
4. Color-critical graphs: definition, connectivity properties, Hajos' contruction and its universality; sparse and dense color-critical graphs; long paths in color-critical graphs.
5. Coloring planar graphs, Heawood's 5-color theorem.
6. Coloring, density, minors and subdivisions: basic definitions; Hadwiger's and Hajos' conjectures; Kostochka-Thomason bound; independent sets and Hadwiger's conjecture, Duchet-Meyniel theorem.
7. Perfect graphs: definition; classes of perfect graphs; chordal graphs; weak perfect graph theorem.
8. List coloring: definition; choosability of complete bipartite graphs; degrees and choice number — upper and lower bounds; choosability in planar graphs, Thomassen's 5-choosability theorem and its tightness.
9. Coloring random graphs: chromatic number of the random graph; list coloring version; performance of the greedy algorithm; Hadwiger's and Hajos' conjectures for random graphs.
10. Coloring locally sparse graphs: graphs with high girth and high chromatic number, Erdős' existence argument and explicit contructions.
11. Hypergraph coloring, property B — lower and upper bounds; applying the Local Lemma.
12. Edge coloring: König's theorem; Vizing's theorem and its extensions, Andersen-Goldberg theorem.

Bibliography

1. Topics in Chromatic Graph Theory, edited by L. W. Beineke and R. J. Wilson, Cambridge University Press, 2015 (available online through our library).
2. Standard graph theory books.