

GRAPH THEORY

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Fall Semester 2005

Course number: 0366-3267

When and where: Wed. 16-19, Schreiber 007.

Prospective audience: the course is intended for second and third year undergraduate students in Mathematics or Mathematics and Computer Science.

Prerequisites: first year courses in mathematics, most notably Discrete Mathematics.

Requirements and grade: Homeworks will be given roughly once every two weeks; submitting at least two thirds of them is mandatory; the final grade will be composed from the final exam's grade (90%) and homeworks (10%).

Syllabus

1. Basic notions: graph, graph isomorphism, adjacency and incidence matrices, paths, cycles, connectivity, subgraphs. Vertex degrees, Sperner's lemma and Brouwer's fixed point theorem.
2. Trees. Equivalent definitions of a tree. Spanning trees. Cayley's formula, matrix-tree theorem.
3. Connectivity. Vertex connectivity and edge connectivity. Degrees and connectivity, Mader's theorem. Blocks, 2-connected graphs. Menger's theorem.
4. Eulerian and Hamiltonian graphs. Theorems of Dirac, Ore, Chvátal-Erdős.
5. Matchings. Theorems of Hall and König for bipartite graphs. Tutte's theorem.
6. Colorings. Vertex colorings, degrees and chromatic number. Greedy algorithm. Brooks' theorem. Color-critical graphs. Graphs with high girth and high chromatic number. Edge coloring, theorems of König and Vizing.
7. Ramsey theory. Ramsey's theorem. Upper and lower bounds for Ramsey numbers. Examples of Ramsey-type theorems.
8. Extremal graph theory. Turán's theorem. Kővari-Sós-Turán theorem.
9. Planar graphs. Euler's formula. Kuratowski's theorem. Coloring planar graphs.
10. Topics (time permitting), to be chosen between: Random graphs and the probabilistic method; Perfect graphs; Graph eigenvalues and tools from linear algebra; Directed graphs.

Bibliography

1. J. A. Bondy and U. S. R. Murty, Graph theory with applications, MacMillan, 1976.
2. L. Lovász, Combinatorial problems and exercises, 2nd edition, North-Holland, 1993.
3. D. West, Introduction to graph theory, 2nd edition, Prentice Hall 2001.