PERCOLATION ON FINITE GRAPHS

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Spring Semester 2026

Course number: 0366.4118.
When and where: Sundays 15-18, Shenkar Physics 105.
Prospective audience: the course is intended for graduate and advanced undergraduate students interested in Combinatorics and Probability.
Prerequisites: working knowledge of graph theory, as provided by standard graph theory courses; close familiarity with basic notions of probability.
Requirements and grade: Homeworks will be given once every two-three weeks, but not graded; final grade will be based solely on in-class exam.

Syllabus

(Subject to changes/detailization as the course progresses)

1. Models of random graphs, $G(n,p)$, $G(n,m)$, random graph processes. Multiple exposure. Thresholds. Comparing $G(n,p)$ and $G(n,m)$.
2. Phase transition in $G(n,p)$: Erdős-Rényi's classical argument for typical component sizes before and after criticality. Interpretation through branching processes.
3. Hitting time of connectedness in the random graph process.
4. Perfect matching in $G(n,p)$.
5. Random subgraphs, model G_p. Subcritical random subgraphs of regular graphs — component sizes. Phase transition and component sizes in the supercritical random hypercube. Polynomial diameter of the giant component in the supercritical random hypercube.
6. Hitting time of connectedness in the random subgraph process for the hypercube.
7. Perfect matching in the random hypercube.
8. Site percolation/vertex-random subgraphs. Component sizes and appearance of the giant component in the supercritical site percolation on the hypercube.
9. Percolation on random regular graphs.

Bibliography

1. B. Bollobás, **Random graphs**, Cambridge Studies in Advanced Mathematics. Cambridge University Press, Cambridge, second edition, 2001.
2. A. Frieze and M. Karoński, **Introduction to random graphs**, Cambridge University Press, Cambridge, 2016. [Updated online version available at the website of Alan Frieze.]
3. S. Janson, T. Łuczak, and A. Ruciński, **Random graphs**, Wiley-Interscience Series in Discrete Mathematics and Optimization. Wiley-Interscience, 2000.