The Tree Model: Stochastic Dominance, Learning and Subjective Option Evaluation

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October 2021

The Objective

- **First part:** Investigate the implications of several stochastic dominance relations among priors in the binomial tree model.
  - Stochastic dominance relation among priors
  - Bayesian learning: stochastic dominance relation among posteriors
  - Stochastic dominance relation of probability distributions over certain histories.
- **Second part:** Examine the implications on *subjective* option valuation.

The Tree Model

Based on John C. Cox, Stephen A. Ross and Mark Rubinstein: *Option pricing: A simplified approach*, 1979.

- Observe a random outcome each period, either **U**, or **D**
- $\mathbb{P}(\mathbf{U}) = p, \quad \mathbb{P}(\mathbf{D}) = 1 - p, \;\; p \in [0, 1]$
- DM does not know p, but holds a prior belief (CDF) F
- A length-t history: h_t

  the frequency of **U** in h_t: $\theta(h_t) = \frac{\#\mathbf{U}}{t}$
- A set of length-t histories: H_t
- The set of all length-t histories: $\mathscr{H}_t$

FOSD and the Tree Model

Definition of FOSD

Consider two priors F and G. We say $F \succeq_{\text{FOSD}} G$, if $F(p) \leq G(p)$ for any p.

An Equivalent Condition for FOSD (well known)

$F \succeq_{\text{FOSD}} G$ if and only if for any increasing function u,

$$\mathbb{E}_F(u) \geq \mathbb{E}_G(u).$$

For any t, F (similarly, G) induces a probability distribution over the $t+1$ frequencies $\frac{0}{t}, \ldots, \frac{t}{t}$ via

$$\mathbb{P}_F(k,t) := \int_0^1 \binom{t}{k} p^k (1-p)^{t-k} \mathrm{d}F, \quad 0 \leq k \leq t.$$

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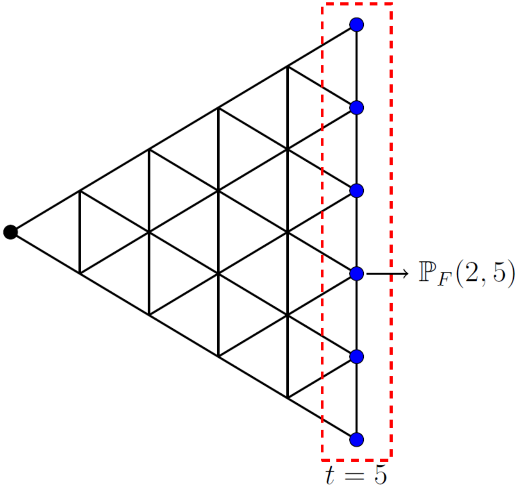
FOSD and the Tree Model



FOSD and the Tree Model

Theorem 1

Consider two prior beliefs F and G about the state of nature p. The following conditions are equivalent.

- $F \succeq_{\text{FOSD}} G$
- for any t, $t = 1, 2, \ldots$, $\{\mathbb{P}_F(k,t)\}_{k=0}^{t} \succeq_{\text{FOSD}} \{\mathbb{P}_G(k,t)\}_{k=0}^{t}$.

Remark. It is well known that FOSD is not preserved under conditioning.

- If $D \subseteq [0,1]$, then $F|D \not\succeq_{\text{FOSD}} G|D$.
- For a history h_t and a set of histories H_t, $F|h_t \not\succeq_{\text{FOSD}} G|h_t$, $F|H_t \not\succeq_{\text{FOSD}} G|H_t$.

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FOSD and the Tree Model

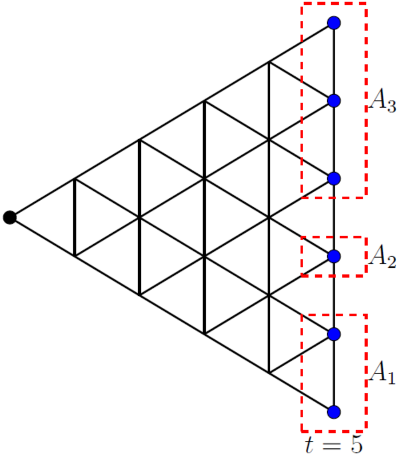

Figure: A Partition of $\mathscr{H}_5$: $\mathscr{P}_5 = \{A_1, A_2, A_3\}$

FOSD and the Tree Model

Corollary

Consider a partition $\mathscr{P}_t = \{A_1, A_2, ..., A_n\}$ of $\mathscr{H}_t$, where $A_1,\ A_2, ..., A_n$ are atoms of the partition s.t. for any $h_t \in A_i$ and for any $h'_t \in A_j$, $\theta(h_t) > \theta(h'_t)$ whenever $i > j$.

Then $F \succeq_{\text{FOSD}} G$ if and only if for any such partition $\mathscr{P}_t$ of $\mathscr{H}_t$, the probability distribution over atoms of $\mathscr{P}_t$ under F first order stochastically dominates that under G.

FOSD and European Options

European Options

A European call/put option gives its holder the right but not obligation to buy/sell a unit of certain asset for a pre-specified price (strike price) at a specific future date (expiration date).

- Expiration date: T.
- Strike price: $\bar{S}$.
- Asset price: $S(k, T)$ at time T, after k times **U**. We assume that it is monotonic in k.
- Value at expiration date T: $\max\{S(k, T) - \bar{S}, 0\}$.

FOSD and European Options

No-Arbitrage Approach

- Replicate exactly the price movement of the underlying asset using existing assets.
- Equilibrium option price must prevent any arbitrage opportunity.
- The prior beliefs play no role in determining the option price.

Subjective Option Valuation

Suppose one cannot replicate the price movement of the underlying asset, say the market is incomplete.
The valuation of an option is now based on a prior beliefs.

FOSD and European Options

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Subjective Option Valuation

Suppose one cannot replicate the price movement of the underlying asset, say the market is incomplete.
The valuation of an option is now based on a prior beliefs.

FOSD and European Options

Theorem 2

Consider two prior beliefs F and G about the state of nature p. Then

- $F \succeq_{\text{FOSD}} G$ if and only if the ex ante value of every European call option is greater under F than under G;
- $F \succeq_{\text{FOSD}} G$ if and only if the ex ante value of every European put option is greater under G than under F.

LR and the Tree Model

Definition – LR

- We say that F *likelihood ratio dominates* G ($F \succeq_{\mathsf{LR}} G$), if $\frac{f(p)}{g(p)}$ is increasing in p. Here f, g are Radon-Nikodým derivatives w.r.t. some adequate measure (e.g., the average of the two, not necessarily the Lebesgue measure).

- Alternative definition in CDFs: For any $x_1 > x_2 > x_3$,

  $$[F(x_1) - F(x_2)][G(x_2) - G(x_3)] \geq [F(x_2) - F(x_3)][G(x_1) - G(x_2)].$$

  It means that the transformation φ s.t. $F(x) = \varphi(G(x))$ is convex.

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  It means that the transformation φ s.t. $F(x) = \varphi(G(x))$ is convex.

Graphical Property of LR

Recall, for any $x_1 > x_2 > x_3$,

$$[F(x_1) - F(x_2)][G(x_2) - G(x_3)] \geq [F(x_2) - F(x_3)][G(x_1) - G(x_2)]$$

or (when well defined)

$$\frac{[F(x_1) - F(x_2)]}{[G(x_1) - G(x_2)]} \geq \frac{[F(x_2) - F(x_3)]}{[G(x_2) - G(x_3)]}.$$

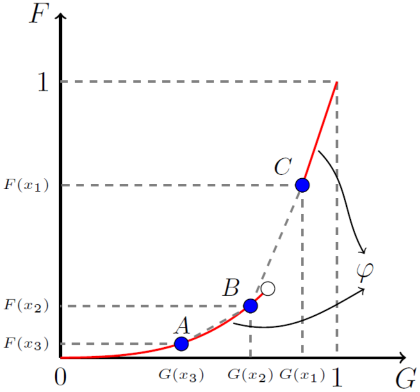

Figure: The transformation φ s.t. $F(x) = \varphi(G(x))$ when $F \succeq_{\mathrm{LR}} G$

LR and the Tree Model

An Equivalent Condition for LR [Lehrer and Wang, 2020]

$F \succeq_{\mathsf{LR}} G$ if and only if

$$\mathbb{E}_F(uv)\mathbb{E}_G(v) \geq \mathbb{E}_G(uv)\mathbb{E}_F(v) \quad \text{or} \quad \frac{\mathbb{E}_F(uv)}{\mathbb{E}_F(v)} \geq \frac{\mathbb{E}_G(uv)}{\mathbb{E}_G(v)}$$

for all functions u and v for which the expectations exist and s.t. v is nonnegative; $u \uparrow$.

Remark. For any v nonnegative, define a new CDF

$$\hat{F}(p) := \frac{\mathbb{E}_F(v\mathbf{1}_{(-\infty,p]})}{\mathbb{E}_F(v)}.$$

The equivalent condition says $\mathbb{E}_{\hat{F}}(u) \geq \mathbb{E}_{\hat{G}}(u)$ for any $u \uparrow$, or $\hat{F} \succeq_{\mathsf{FOSD}} \hat{G}$.

LR and the Tree Model

Theorem 5—LR and the Tree Model

Suppose F and G are non-degenerate at 0 or 1. The following conditions are equivalent:

1. $F \succeq_{\mathsf{LR}} G$.
2. For any set of length-t histories H_t, $F|H_t \succeq_{\mathsf{LR}} G|H_t$.
3. For any H_t, $F|H_t \succeq_{\mathsf{FOSD}} G|H_t$.
4. For any H_t, the probability distribution over H_t induced by F FOSD the probability distribution over H_t induced by G.

LR and the Tree Model

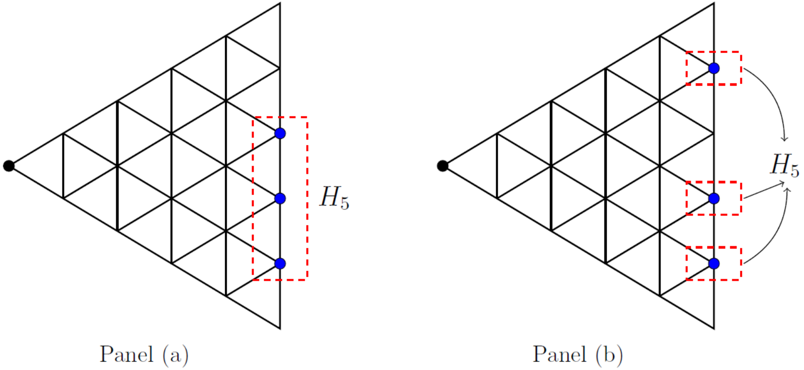

Figure: Histories under consideration for LR

LR and American Options

American Options

An American call/put option gives its holder the right but not obligation to buy/sell a unit of a certain asset for a pre-specified price (strike price) *at or before* a specific future date (expiration date).

- Expiration date: T.
- Strike price: $\bar{S}$.
- Asset price: $S(k,t)$. Assume, $S(k,t)$ is monotone in k for each t.
- Optional exercise payoff: $\max\{S(k,t) - \bar{S}, 0\}$.
- Value function for call option: $V_F^T(k,t)$.

$$V_F^T(k,t) = \max\Big\{S(k,t) - \bar{S},$$
$$\delta\left[\mathbb{P}_F(\mathbf{U}|(k,t))V_F^T(k+1,t+1) + \mathbb{P}_F(\mathbf{D}|(k,t))V_F^T(k,t+1)\right]\Big\}.$$

LR and American Options

Proposition 1

Suppose F and G are not degenerate at 0 or 1 and that $F \succeq_{\mathsf{LR}} G$. Then $V_F^T(k,t) \geq V_G^T(k,t)$ for every (k,t).

Proposition 2

Suppose F and G are not degenerate at 0 or 1 and that $F \succeq_{\mathsf{LR}} G$. Then for every American call option, whenever it is also optimal to exercise under F, it is optimal to exercise under G.

LR and American Options

Proposition 1

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Proposition 2

Suppose F and G are not degenerate at 0 or 1 and that $F \succeq_{\mathsf{LR}} G$. Then for every American call option, whenever it is also optimal to exercise under F, it is optimal to exercise under G.

LR and American Options

Theorem 6

Consider two prior beliefs F and G. $F \succeq_{\mathsf{LR}} G$ if and only if for every American call option and every set of histories H_t s.t. the option is not exercised, the expected option value conditional on H_t is greater under F than under G.

HR, RHR and the Tree Model

Definition—HR, RHR

- We say that F *reverse hazard rate dominates* G ($F \succeq_{\mathsf{RHR}} G$), if for any p,
$$\frac{f(p)}{F(p)} \geq \frac{g(p)}{G(p)}.$$
- We say that F *hazard rate dominates* G ($F \succeq_{\mathsf{HR}} G$), if for any p,
$$\frac{f(p)}{1-F(p)} \leq \frac{g(p)}{1-G(p)}.$$

Remark. As before, f and g are Radon-Nikodým derivatives w.r.t. some measure (not necessarily the Lebesgue measure. The distributions might have atoms).

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Remark. As before, f and g are Radon-Nikodým derivatives w.r.t. some measure (not necessarily the Lebesgue measure. The distributions might have atoms).

A different look at HR, RHR

Alternative Definition of HR, RHR

- $F \succeq_{\text{RHR}} G$, if and only if for any $p_H > p_L$,

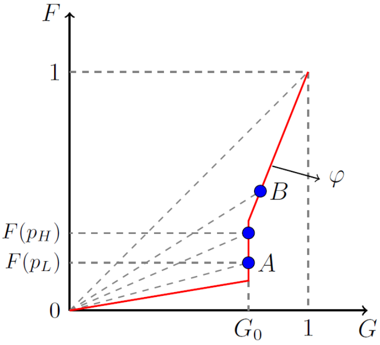
$$F(p_H)G(p_L) \geq G(p_H)F(p_L) \quad \text{or} \quad \frac{F(p_H)}{G(p_H)} \geq \frac{F(p_L)}{G(p_L)};$$

- $F \succeq_{\text{HR}} G$, if and only if for any $p_H > p_L$,

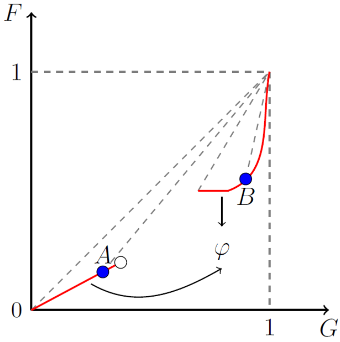
$$\begin{aligned} (1 - F(p_H))(1 - G(p_L)) \quad &\geq \quad (1 - G(p_H))(1 - F(p_L)) \\ \text{or} \quad \frac{1 - F(p_H)}{1 - G(p_H)} \quad &\geq \quad \frac{1 - F(p_L)}{1 - G(p_L)}. \end{aligned}$$

Graphical Property of HR, RHR

- $\varphi : [0,1] \rightrightarrows [0,1]$ is the transformation s.t. $F(p) = \varphi(G(p))$.
- Recall, $F \succeq_{\text{RHR}} G$, if and only if for any $p_H > p_L$, $\frac{F(p_H)}{G(p_H)} \geq \frac{F(p_L)}{G(p_L)}$.



Panel (a): $F \succeq_{\text{RHR}} G$



Panel (b): $F \succeq_{\text{HR}} G$

HR, RHR and the Tree Model

An Equivalent Condition for RHR

$F \succeq_{\text{RHR}} G$ if and only if

$$\mathbb{E}_F(uv)\mathbb{E}_G(v) \geq \mathbb{E}_G(uv)\mathbb{E}_F(v) \quad \text{or} \quad \frac{\mathbb{E}_F(uv)}{\mathbb{E}_F(v)} \geq \frac{\mathbb{E}_G(uv)}{\mathbb{E}_G(v)}$$

for all functions u and v for which the expectations exist and s.t. v is nonnegative, $\downarrow$; $u \uparrow$.

Remark. For any v nonnegative, $\downarrow$, define a new CDF

$$\hat{F}(p) := \frac{\mathbb{E}_F(v\mathbf{1}_{(-\infty,p]})}{\mathbb{E}_F(v)}.$$

The equivalent condition says $\mathbb{E}_{\hat{F}}(u) \geq \mathbb{E}_{\hat{G}}(u)$ for any $u \uparrow$, or equivalently, $\hat{F} \succeq_{\text{FOSD}} \hat{G}$.

HR, RHR and the Tree Model

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RHR – An Equivalent Condition

An Equivalent Condition for HR

$F \succeq_{\mathsf{HR}} G$ if and only if

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for all functions u and v for which the expectations exist and s.t. v is nonnegative, $\uparrow$; $u \uparrow$.

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RHR and the Tree Model

For any t and any $\theta \in [0,1]$, define $H_t^-(\theta) := \{h_t \in \mathscr{H}_t | \theta(h_t) \leq \theta\}$.

Theorem 3 – RHR and the Tree Model

Suppose F and G are non-degenerate at 0 or 1. The following conditions are equivalent:

1. $F \succeq_{\text{RHR}} G$;
2. For any t and any $\theta \in [0,1]$, the posterior beliefs satisfy $F|H_t^-(\theta) \succeq_{\text{RHR}} G|H_t^-(\theta)$.
3. For the same set $H_t^-(\theta)$, $F|H_t^-(\theta) \succeq_{\text{FOSD}} G|H_t^-(\theta)$.
4. Consider the same set $H_t^-(\theta)$. The probability distribution over $H_t^-(\theta)$ induced by F FOSD the probability distribution over $H_t^-(\theta)$ induced by G.

A Sketch of the Proof – I

We show that $F \succeq_{\mathsf{RHR}} G \;\Rightarrow\; F|H_t^-(\theta) \succeq_{\mathsf{RHR}} G|H_t^-(\theta)$.

Denote by $B(H_t^-(\theta);p)$ the probability of observing $H_t^-(\theta)$ when the state is p. It is decreasing in p.

$$\left[F|H_t^-(\theta)\right](\bar{p}) = \frac{\mathbb{E}_F(B(H_t^-(\theta);p)\ \mathbf{1}_{[0,\bar{p}]})}{\mathbb{E}_F(B(H_t^-(\theta);p))}, \text{where } B(H_t^-(\theta);p) \downarrow \text{in } p.$$

It follows from the equivalent condition for RHR that

$$\begin{aligned}
\frac{\mathbb{E}_{F|H_t^-(\theta)}(u(p)v(p)))}{\mathbb{E}_{F|H_t^-(\theta)}(v(p))} &= \frac{\mathbb{E}_F\left(u(p)v(p)B(H_t^-(\theta);p)\right)}{\mathbb{E}_F\left(v(p)B(H_t^-(\theta);p)\right)} \\
&\geq \frac{\mathbb{E}_G\left(u(p)v(p)B(H_t^-(\theta);p)\right)}{\mathbb{E}_G\left(v(p)B(H_t^-(\theta);p)\right)} \\
&= \frac{\mathbb{E}_{G|H_t^-(\theta)}(u(p)v(p)))}{\mathbb{E}_{G|H_t^-(\theta)}(v(p))}.
\end{aligned}$$

A Sketch of the Proof – II

We show now that $F|H_t^-(\theta) \succeq_{\text{FOSD}} G|H_t^-(\theta) \;\Rightarrow\; F \succeq_{\text{RHR}} G$.

Recall, $H_t^-(p_H) := \{h_t \in \mathscr{H}_t | \theta(h_t) \le p_H\}$

Lemma

Let $p_L \le p_H$. Then,

$$\lim_{t\to\infty} \left[F|H_t^-(p_H)\right](p_L) = \frac{F(p_L)}{F(p_H)}, \;\; \lim_{t\to\infty} \left[G|H_t^-(p_H)\right](p_L) = \frac{G(p_L)}{G(p_H)}.$$

Suppose $F \not\succeq_{\text{RHR}} G$. There exist $p_L,\ p_H$ with $p_L < p_H$ s.t.

$$\frac{F(p_H)}{G(p_H)} < \frac{F(p_L)}{G(p_L)}, \quad \text{or} \quad \frac{F(p_L)}{F(p_H)} > \frac{G(p_L)}{G(p_H)}.$$

Consider $H_t^-(p_H)$ for sufficiently large t. The lemma together with the fact that $F|H_t^-(p_H) \succeq_{\text{FOSD}} G|H_t^-(p_H)$ yields a contradiction.

HR and the Tree Model

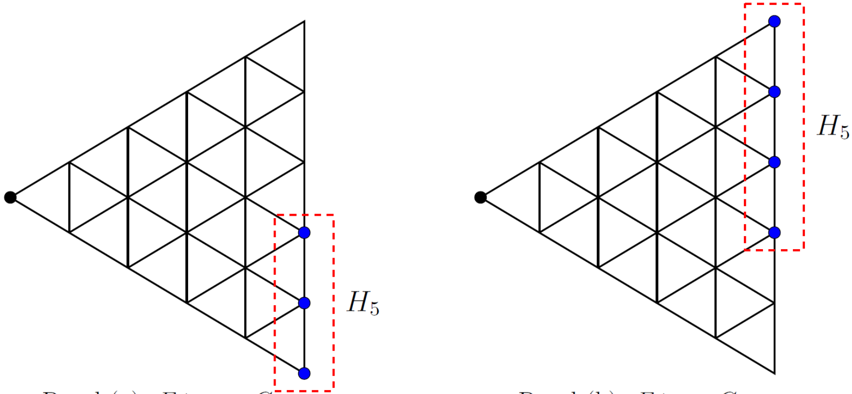
For any t and any $\theta \in [0, 1]$, define $H_t^+(\theta) := \{h_t \in \mathscr{H}_t | \theta(h_t) \geq \theta\}$.

Theorem 3' – HR and the Tree Model

Suppose F and G are non-degenerate at 0 or 1. The following conditions are equivalent:

1. $F \succeq_{\text{HR}} G$.
2. For any t and any $\theta \in [0, 1]$, the posterior beliefs satisfy $F | H_t^+(\theta) \succeq_{\text{HR}} G | H_t^+(\theta)$.
3. For the same set $H_t^+(\theta)$, $F | H_t^+(\theta) \succeq_{\text{FOSD}} G | H_t^+(\theta)$.
4. The probability distribution over $H_t^+(\theta)$ induced by F FOSD the probability distribution over $H_t^+(\theta)$ induced by G.

HR, RHR and the Tree Model



Panel (a): $F \succeq_{RHR} G$ Panel (b): $F \succeq_{HR} G$

HR, RHR and European Options

Theorem 4

Suppose F and G are non-degenerate at 0 or 1.

- $F \succeq_{\mathsf{HR}} G$ if and only if for every European *call* option, the option value conditional on the option is exercised is greater under F than under G.
- $G \succeq_{\mathsf{RHR}} F$ if and only if for any European *put* option, the option value conditional on the option is exercised is greater under F than under G.

HR, RHR and European Options

Theorem 4

Suppose F and G are non-degenerate at 0 or 1.

- $F \succeq_{\text{HR}} G$ if and only if for every European *call* option, the option value conditional on the option is exercised is greater under F than under G.
- $G \succeq_{\text{RHR}} F$ if and only if for any European *put* option, the option value conditional on the option is exercised is greater under F than under G.

Same Prior, Different Histories

Proposition 3

Let h_t and h'_t be two histories of length t. If h_t has more observations of **U**, then for any prior belief $F(p)$ over $[0, 1]$, $F|h_t \succeq_{\text{FOSD}} F|h'_t$.

Corollary

Let F be any non-degenerate prior belief. Consider two sets of histories H_t and H'_t. Suppose the frequency of any history in H_t is no less than the frequency of any history in H'_t, then $F|H_t \succeq_{\text{FOSD}} F|H'_t$.

Same Prior, Different Histories

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Same Prior, Different Histories – European Options

- Fix a prior belief, consider $h_t,\ h_t'$ with $\theta(h_t) > \theta(h_t')$ (note, the same t). Then, for any European call option, the option value conditional on h_t is greater than the option value conditional on h_t'.

- Proposition 3 does not hold for histories of different lengths: if $\theta(h_t) > \theta(h_{t'})$, where $t \neq t'$, then typically, $F|h_t \not\succeq_{\text{FOSD}} G|h_{t'}$.

Same Prior, Different Histories – European Options

- Fix a prior belief, consider $h_t,\ h'_t$ with $\theta(h_t) > \theta(h'_t)$ (note, the same t). Then, for any European call option, the option value conditional on h_t is greater than the option value conditional on h'_t.

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Summary

- We analyze the implications of different stochastic dominance relations in the tree model.
- We show the connection between the priors and the posteriors as far as stochastic dominance is concerned.
- Most importantly, we establish the connection between subjective option valuation (European and American, call and put) and various notions of stochastic dominance.