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Subjective utilitarianism: Individual decisions in a social context ☆

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Abstract

Individual decisions are often subjectively affected by other-regarding considerations. We model a decision maker influenced by a grand group of significant others. Each sub-group of significant others is a possible *social context*, and the decision maker has different preferences in different social contexts. An axiomatic characterization of such preferences is offered. The characterized representation takes a simple *subjective utilitarian* form: (a) the decision maker ascribes to each significant other a utility function, representing the decision maker's subjective perception of this other person's tastes, and (b) in any specific social context the decision maker evaluates alternatives by adding together her or his own personal utility and the sum of all group members' utilities as subjectively perceived by the decision maker.

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1. Introduction

It is widely recognized in the economic literature that people may have other-regarding preferences, namely preferences that are not strictly selfish, but also depend on others in their society. People may be inequity averse, care about social welfare, or be sensitive to their social status, to name only a few examples. The idea that preferences and decisions of an individual may be affected by others' payoff was demonstrated in diverse experiments (see Cooper and Kagel (2013) for a survey). On the theoretical side, various models were designed to describe and explore forms of other-regarding preferences. Some examples include Fehr and Schimdt (1999) and Bolton and Ockenfels (2000), who offer models in which agents are inequity averse; Charnes and Rabin (2002) who present experiments and a functional form that incorporates social welfare concerns into agents' preferences; and Dufwenberg et al. (2011), in which general equilibria is explored based on individuals' other-regarding preferences that depend on all agents' opportunities.[1]

In the majority of models of individual preferences that exhibit other-regarding features, concern for the welfare of others is incorporated into individual preferences in one of two ways. In some models, an individual's evaluation of the welfare of others is done using one utility, either the personal utility of that modeled individual, or some social, universal utility. Such models are used to describe, for instance, an individual's concern for fairness, which is measured through this individual's eyes or according to some social norm. These models cannot be used to describe an individual assessing welfare of different others in different ways. Namely, they cannot be used to model an individual who is sensitive to heterogeneous tastes of others.

In other models in the other-regarding literature, welfare of others is evaluated using their actual utilities. That way, heterogeneous tastes of others are taken into account. But these models require considerably more observables than models of the first type. Whereas models of the first type require to observe only the preferences of a single individual, models of the second type assume that the preferences of all other individuals are available as well. Models of the second type are employed, for example, when equilibria is investigated under an assumption of other-regarding preferences of agents.

We offer a third way in which concern for others' welfare may enter the preferences of an individual. Our model describes an individual who attempts to account for others' heterogeneous tastes. Yet, the model does not require to observe more than the preferences of the single individual. The key is that we take a subjective approach.

Before getting into the specifics of the model, we motivate the discussion with two examples. First, imagine a person buying takeout for dinner with friends. This person may personally prefer Italian to Chinese food, but settle for Chinese food nevertheless, since his friends prefer it to Italian. Similarly, consider someone booking a family vacation. She may personally rather travel to Paris than to an all-inclusive resort, yet opt for the resort if her children like the resort better.

In both examples, an individual choosing one alternative over another takes into account the tastes of others that will be affected by this decision. The choice itself (the purchase of takeout food, the booking of a vacation) is made by the individual alone, however consumption of the chosen commodities is shared with a group of significant others (the decision maker's friends, the decision maker's family). We argue that in cases of this sort, an individual's decision is affected by the group with which the individual consumes (the group of friends attending dinner affects

[1] For an extensive discussion of other-regarding models see Postlewaite (2010).

the decision which takeout food to buy, family members affect the decision which vacation to book). Thus we may see an individual preferring one alternative over another on one occasion, but exhibiting the reversed preference on another occasion, where this reversal is a consequence of considering consumption with different groups of people, rather than an expression of indifference or inconsistency.

The model proposed in this paper is designed to describe decisions of the sort outlined above. Within the model, the preferences of a single individual are considered. Groups of others with which this individual consumes are made explicit, and taken into account in the individual's preference. This allows us to accommodate a decision maker who is affected by such groups, possibly exhibiting reversals of preferences over goods, resulting from considering different reference groups.

Formally, one of the primitives assumed is a grand group of significant others, which is meant to include all those people who may affect the decision maker's preferences. Consumption is possible with any sub-group of this grand group, and each of these sub-groups is called a *social context*, or a *group context*. Alternatives take social contexts into account, in that the individual preferences considered are over pairs of a lottery and a social context with which the lottery is to be consumed. The setup is therefore a classic von-Neumann and Morgenstern one (vNM; von Neumann and Morgenstern, 1944), supplemented with groups of referent others. With this setup it is possible to describe, for instance, an individual whose strict preference is to spend a vacation with her spouse in Paris rather than in an all-inclusive resort, but exhibits the reversed preference, for the resort over Paris, when considering a vacation with the entire family. Moreover, this setup allows us to address dual questions, such as, whether the decision maker prefers to spend a vacation in Paris with her spouse alone, to spending it with both her spouse and her kids. In fact, these types of comparisons, in which one component in the compared alternatives is kept constant, are the only types of comparisons which are assumed to be observed. We prove that with these limited comparisons, other comparisons, involving different lotteries as well as different groups, uniquely follow.

A possible critique at this point would be, that a vacation with one's spouse in Paris, and a vacation with one's entire family in Paris, are simply two different alternatives. They can be modeled as such in an abstract manner, without imposing a structure containing a social context. Our reply is that an explicit modeling of a social context allows us not only to distinguish between two such alternatives, but moreover to ask if and how the decision maker's evaluation of these alternatives differs as a function of the social context involved. And indeed our aim is to identify a systematic dependency of preferences on a social context. To compare, vacation today and vacation tomorrow are also two different alternatives. However, specifying time as part of an alternative's description facilitates a characterization of time discounted preferences.

Having a setup which supports an investigation of preferences depending on a social context, the question still remains, what kind of dependency does the model depict? We suggested above that people take into account others' tastes when forming a decision. On the other hand, we insisted on a model that relies on observing the preferences of a single individual. But without observing the preferences of others, it is impossible to extract their tastes. So how can their tastes be taken into account?

To answer this question, we return to the examples above. These examples describe a person buying Chinese food rather than Italian food on the premise that his friends prefer Chinese, and another person booking a family vacation in an all-inclusive resort based on the view that her kids prefer it to a vacation in Paris. Decisions in both cases are made by one person alone. Relevant others do not actively participate in making the decision, but rather indirectly affect it through the decision maker's perception of their preferences. This is precisely what our model aims to describe. A single decision maker, whom at his or her own discretion determines how to incorporate others' preferences. Consequently, any other-regarding effect on the preferences of the decision maker is entirely subjective, dependent upon the decision maker's perception of others' tastes, as well as on his or her inclination to be considerate of those tastes. Accordingly, the representation offered does not depend on others' true tastes (which are not observed), but on the way those tastes are subjectively perceived by the decision maker. Note that as a result tastes of others that are incorporated into the decision maker's preferences may contradict their actual tastes. We remark on this issue in Subsection 3.1.2 and discuss it in more details in Supplementary Appendix B.

In line with the above subjective dependence on others' tastes, the representation characterized depends on vNM utility functions, one per significant other. These are interpreted as representing others' preferences as subjectively perceived by the decision maker. The representation aggregates these utilities, and a vNM utility representing the decision maker's personal preferences, in a *subjective utilitarian* manner. That is to say, a lottery in a group context is evaluated through the utilitarian sum of the decision maker's personal vNM utility from this lottery, and the vNM utilities from this lottery of all group members, as subjectively perceived by the decision maker.

The work in the paper is axiomatic, behaviorally characterizing subjective utilitarianism. The main assumptions in the identification of a subjective utilitarian decision maker require that the effect of considering social contexts be consistent, both when considering groups with shared members, and in the preference reversals that are inflicted. Preference reversals are interpreted as representing compromises made by the decision maker in favor of significant others.

Formally, a pair (p, G) of a lottery p in the context of a group G is evaluated by,

$$V(p, G) = u_0(p) + \sum_{j \in G} v_j(p), \tag{1}$$

where u_0 is the personal vNM utility of the decision maker, and v_j is the vNM utility that the decision maker subjectively ascribes to individual j.

Importantly, the decision maker attributes to each referent other j one vNM utility that is used whenever a lottery is evaluated in the context of a group containing j. We interpret this utility as reflecting the tastes of individual j, as these are perceived by the decision maker. The representation theorem furthermore delivers that utilities ascribed to others are calibrated relative to the decision maker's own utility. That is to say, others' utilities are unique once the decision maker's personal utility is fixed. Therefore, along with others' tastes the theorem derives the extent to which the decision maker wishes to comply with these tastes.

The representation suggested here constitutes a subjective version of Harsanyi's utilitarianism (Harsanyi, 1955). Here as well the tastes of all individuals in society are taken into account in an additive manner. However, while the choice of weights in Harsanyi's model poses difficult ethical questions regarding interpersonal utility comparisons in a society, in the model discussed here the preferences at issue are of a single individual, therefore no ethical dilemma arises. The calibration of others' utilities within the subjective utilitarian model simply reflects the degree to which the decision maker wishes to be considerate of others' tastes.

Before turning to a discussion of relevant literature, observe that although our interpretation of the model is of an individual making decisions in consideration of others' welfare, alternative interpretations may apply. For instance, the decision maker may be driven by spite. Or, the decision maker may be paternalistic, basing decisions on her or his own judgment of what is better for significant others. Yet, whatever the correct interpretation, our model characterizes an other-regarding decision maker, on whose decisions every referent other has an additive effect, the same across different groups.

1.1. Related literature

Most closely related to our work are axiomatic papers which characterize other-regarding representations of a single decision maker. In all these models that we are aware of, however, welfare of others is evaluated according to one of the methods specified above. That is to say, either the evaluation is based on the same utility for all others involved (this may be the decision maker's personal utility, or a utility representing some social measure), or it is based on observing true utilities of others.

Within the second approach, Segal and Sobel (2007) characterize an individual in a strategic setting, whose preferences over her or his own strategies (given fixed strategies of others) admit a utilitarian representation. A player's utilitarian evaluation of a strategy weighs the player's own utility from that strategy (given the others' strategies), together with the other players' utilities from the profile considered. The characterization assumes that the utilities from outcomes of all players are observable. In a non-strategic setup, Fershtman and Segal (2018) investigate preferences of individuals in a society, who are influenced by each others' tastes. They assume observability of all individuals' preferences, and describe an equilibrium where the observable preferences of an individual depend on her or his core personal preferences, and on the observable preferences of all others in society. Fershtman and Segal characterize axiomatically such equilibrium preferences where the dependence on others' preferences is only through their average observable preferences (i.e., average of the corresponding utilities).

Among the axiomatic models that belong to the first approach is Maccheroni et al. (2012), studying individual preferences over allocations of general acts to sub-groups of agents. The decision maker described compares her or his allocated acts to others' allocated acts through a personal utility, or by applying a social value function, the same for all agents. The representations in that paper thus characterize a decision maker who is sensitive to social status. Another axiomatic paper involving an individual with social status concerns is Ok and Koçkesen (2000), describing a preference over income distributions which representation reflects the individual's wish to occupy a higher status than others in society.

A different motivation is expressed in Karni and Safra (2002). These authors describe an individual choice behavior which is driven by ethical motives in addition to the more traditional purely-selfish motives. The paper offers a representation which involves one function representing the purely selfish preferences of a decision maker, and another one that represents his or her moral preferences. In another paper, Dillenberger and Sadowski (2012) characterize an individual decision maker who chooses between menus of payoff allocations to himself or herself and another individual, motivated by a tradeoff between self interest and a wish not to appear selfish.

The paper is organized as follows. Section 2 describes the setup and conditions that lead to a basic representation. Section 3 discusses the subjective utilitarianism axioms and the main result. All the proofs appear in Section 4. Some additional comments can be found in supplementary appendices.

2. Setup and basic representation

Suppose a finite set of prizes X, and a set Y of lotteries over X, namely probability distributions over X with finite support. The paper examines the preferences of an individual who operates in a group context. The individual whose preferences are examined is called 'Individual Zero', and the grand set of referent individuals is $I = \{1, \ldots, N\}$, each $i = 1, \ldots, N$ being a 'significant other' for Individual Zero. A *social context*, or a *group context* is a subset $G \subseteq I$. Consuming lottery p when Individual Zero is with a group G, which we refer to as consuming p in a context G, is denoted (p, G). Consuming p alone is written simply as p (shortening $(p, \emptyset)$). Individual Zero's preferences, denoted $\succsim$, are over such pairs of lotteries and contexts, that is, $\succsim \subseteq (Y \times 2^I)^2$. The symmetric and asymmetric components of $\succsim$ are respectively denoted $\sim$ and $\succ$.

Our first step in characterizing subjective utilitarianism is to identify when the preferences of Individual Zero, both within and across contexts, are represented by a family of vNM utilities. The first assumption in doing so states that there are two purely private prizes, strictly ranked when consumed alone, however their consumption is unaffected by a social context. For instance, Individual Zero may personally prefer a peppermint chewing gum to a raspberry chewing gum, but may not care if he or she chews the gum with or without a group.[2]

B1. Individualistic ranking.
There are $r^*, r_* \in X$ such that $r^* \succ r_*$, and for any group G, $(r^*, G) \sim r^*$ and $r_* \sim (r_*, G)$.

The second assumption in the basic characterization states that the decision maker can determine simple comparisons, in which alternatives share a common component. That is to say, the decision maker is able to compare the consumption of two different lotteries with the same group, or the consumption of the same lottery with two different groups. For instance, the decision maker can determine whether she prefers a family vacation in Paris to a family vacation in a resort, as well as whether she prefers a vacation in Paris with her spouse to a vacation in Paris with the entire family.

B2. Completeness over simple comparisons.
Let p, q be lotteries and G, H groups. Then:

(a) Either $(p, G) \succsim (q, G)$ or $(q, G) \succsim (p, G)$.
(b) Either $(p, G) \succsim (p, H)$ or $(p, H) \succsim (p, G)$.

Supposing three additional axioms, detailed below, we are able to show that more complex comparisons, say, between traveling to Paris with one's spouse and spending a family vacation in a resort, are implied. These three axioms are standard transitivity, archimedeanity, and independence, adapted to our framework.

B3. Transitivity.
For any three lotteries p, q and r, and any three groups G, H and K, if $(p, G) \succsim (q, H)$ and $(q, H) \succsim (r, K)$ then $(p, G) \succsim (r, K)$.

[2] Note that our model considers only Individual Zero's preferences and perception of others. We have no way of knowing whether other individuals care, and if so in what way, about Individual Zero's consumption of prizes. For example, another individual may hate the chewing noise that Individual Zero makes, but as long as this is not recognized by the latter, it has no effect on observed preferences.

B4. Archimedeanity within contexts.
Let p, q, r be lotteries and G a group. If $(p, G) \succ (q, G) \succ (r, G)$ then there are $\alpha, \beta \in (0, 1)$ such that $(\alpha p + (1-\alpha)r, G) \succ (q, G) \succ (\beta p + (1-\beta)r, G)$.

B5. Independence.
Let p, q and r be lotteries, and G and H groups. If $(r, G) \sim (r, H)$ then,

$$(p, G) \succsim (q, H) \iff (\alpha p + (1-\alpha)r, G) \succsim (\alpha q + (1-\alpha)r, H) \ , \ \alpha \in (0, 1) \ .$$

Independence implies, in particular, vNM's independence within each fixed context. As weak order and Archimedeanity within each context are stated in B2 and B4, respectively, a within-context vNM representation immediately ensues. In the proposition that follows it is stated that axioms B1-B5 are equivalent to a vNM representation of $\succsim$, such that the vNM utilities derived per context furthermore serve to compare the consumption of lotteries across different contexts. In particular, completeness of $\succsim$ over all possible pairs of lottery and context is implied.

Proposition 1. *Let $\succsim$ be a binary relation over $Y \times 2^I$. Then B1-B5 are satisfied, if and only if, there exist vNM utility functions u_G, $G \subseteq I$, such that for any lotteries p and q and groups G and H,*

$$(p, G) \succsim (q, H) \iff u_G(p) \geq u_H(q) \ .$$

Furthermore, these utilities are unique up to joint shift and scale,[3] *and there exist two lotteries, r^* and r_*, such that for any group G, $u_G(r^*) = u_\emptyset(r^*) > u_\emptyset(r_*) = u_G(r_*)$.*

The proof appears in Section 4. It begins with finding equivalents to lotteries in all social contexts in terms of r^* and r_* when consumed alone. These equivalents are then employed to extend the within-context vNM representation to comparisons across different contexts.

3. Subjective utilitarianism

Following Proposition 1, Individual Zero applies a family of vNM utilities, one per group of significant others, to compare lotteries both within and across contexts. To further obtain that these utilities take a subjective utilitarian form, four additional axioms are imposed. The first, termed *Compromise*, addresses reversals of personal preferences when Individual Zero is joined by groups of referent others. Compromise states that if a personal preference is reversed in the context of group G as well as in the context of a disjoint group H, then it is also reversed when consumption is made with members of G and H together. The axiom supports the interpretation that reversal of personal preferences is the result of compromising with others whose preferences are opposite. For if opposite preferences of members of G are strong enough to make Individual Zero reverse a personal preference, and the same holds separately for H, then the preferences of members of G and H together give even more of an incentive to compromise.

Compromise eliminates the possibility of cross-effects when referent individuals are joined together. For example, it rules out a decision maker who, when alone, prefers staying home to

[3] That is to say, if $\hat{u}_G$, $G \subseteq I$, is any other array of utilities representing $\succsim$ in the same manner, then there are $\sigma > 0$, τ such that, $\hat{u}_G = \sigma u_G + \tau$, for every G. For $G = \emptyset$, the utilities u_G and $\hat{u}_G$ are the utility functions of Individual Zero alone.

going to the movies, when with her spouse or with a friend prefers the movies to staying home, but when with both of them would rather stay home (say because they don't get along).

S1. Compromise.
Let p, q be lotteries and G, H disjoint groups. Then

$$q \succsim p\ ,\ (p, G) \succsim (q, G)\ ,\ (p, H) \succsim (q, H) \implies (p, G \cup H) \succsim (q, G \cup H)\ .$$

Moreover, if any of the three conditions holds strictly then the conclusion is strict as well.

Another form of consistency is imposed on the individual preference, whereby a decision whether to consume a lottery with one group or another is not affected by members belonging to both groups. For an example suppose that Individual Zero prefers to go to a classical concert with one friend rather than with another. Then this axiom postulates that Individual Zero would also prefer going to the concert with his spouse and the first friend, to going with his spouse and the second friend. The motivation is that the first preference is presumably because the first friend likes classical music better than the second, so when comparing two alternatives where the spouse goes anyway, preference is still determined by the friends' liking of classical music. This condition again rules out cross effects between referent others, similarly to the previous assumption.

S2. Consistent influence.
For every three pairwise-disjoint groups, G, H and K, and every lottery p,

$$(p, G) \succsim (p, H) \iff (p, G \cup K) \succsim (p, H \cup K)\ .$$

Lastly, we impose two richness conditions. The first condition requires the existence of a socially consensual lottery, a lottery which the decision maker always prefers to consume with a significant other to consuming it alone. Similarly, this condition also supposes the existence of a socially undesirable lottery, which the decision maker always prefers to consume alone.

S3. Social desirable and undesirable lotteries.
There exist lotteries q^*, q_* such that $(q^*, i) \succ q^*$ and $q_* \succ (q_*, i)$ for every $i \in I$.

Together with repeated applications of Consistent Influence this assumption implies that the same social preferences hold for every nonempty group G: $(q^*, i) \succ q^*$ if and only if $(q^*, \{i, j\}) \succ (q^*, j)$, and since $(q^*, j) \succ q^*$ then $(q^*, \{i, j\}) \succ q^*$, and so on until it is established that $(q^*, G) \succ q^*$. Similarly, $q_* \succ (q_*, G)$.

The second richness condition requires minimal disagreement among reference individuals, so that it is not the case that all of them agree on everything.

S4. Minimal disagreement.
There are $i, j \in I$ and lotteries p, q such that $(p, i) \succsim (q, i)$ but $(q, j) \succ (p, j)$.

Our main theorem states that axioms B1-B5, together with S1-S4, are equivalent to a subjective utilitarian representation: Individual Zero ascribes to each referent individual a subjective vNM utility function, and evaluates a lottery in a group context by adding to his or her own personal utility the sum of the subjective utilities of all group members. Importantly, the utility of each referent individual is the same in all groups to which this individual belongs. Moreover, once the utility of Individual Zero is calibrated, all other utilities are fixed. In other words, the weight that Individual Zero assigns to the preferences of every referent individual is derived endogenously within the model.

Theorem 1. *Let $\succsim$ be a binary relation over $Y \times 2^I$. Then the following two statements are equivalent:*

(i) *Assumptions B1-B5, and S1-S4, are satisfied.*
(ii) *There exist vNM utilities, $u_0, v_1, \ldots, v_N$, such that for any lotteries p and q and groups G and H,*

$$(p, G) \succsim (q, H) \iff u_0(p) + \sum_{j \in G} v_j(p) \geq u_0(q) + \sum_{j \in H} v_j(q) \tag{2}$$

Furthermore, $u_0, v_1, \ldots, v_N$ are unique up to a joint scale and a shift of u_0, and satisfy the following conditions:
(a) *There are lotteries r^* and r_* such that $u_0(r^*) > u_0(r_*)$, and $v_i(r^*) = v_i(r_*) = 0$, for every $i \in I$.*
(b) *There are lotteries q^* and q_* such that $v_i(q^*) > 0 > v_i(q_*)$, for every $i \in I$.*
(c) *There are $i, j \in I$ such that $v_i \neq cv_j$ for any $c > 0$.*

Outcomes r_* and r^* described in (a) can serve as a threshold for the decision maker, as these are outcomes towards which others are indifferent according to the decision maker. It follows that for a lottery p, the decision maker will be (weakly) better off if and only if a referent other j with $v_j(p) \geq v_j(r_*)$ joins the social context in which p is consumed.

3.1. Remarks

3.1.1. Assuming only compromise (S1)

If we assume only Compromise (S1), and omit Consistent Influence (S2), Social desirable and undesirable lotteries (S3), and Minimal Disagreement (S4), we obtain that for every nonempty group T, $u_T = u_0 + \sum_{i \in T} \lambda_i^T v_i$, $\lambda_i^T > 0$ for every $i \in T$. That is to say, lotteries in a social context are evaluated through the utilitarian sum of Individual Zero's utility and a weighted sum of the subjective utilities that Individual Zero subjectively ascribes to others. This results from a simple application of Harsanyi's theorem (see the proof for details). However under S1 alone the weight that is placed on the utility of a referent individual may change depending on the context that is considered. For instance, in one context where both referent individuals i and j are included the weight on j's utility may be larger than that on i's utility, while in another context the opposite may hold.

3.1.2. Perceived vs actual tastes

Utilitarianism in our model is purely subjective. That is to say, the expression of others' tastes in the representation depicts the decision maker's perception of these tastes and not necessarily their actual tastes. The model cannot determine whether the decision maker's perception it true, as preferences of others are not observed. In Supplementary Appendix B we discuss cases where others' actual preferences may be observed, and formulate conditions for the coincidence of perceived and true tastes. Perhaps not surprisingly, such coincidence is essentially a result of a Pareto-type condition.

3.1.3. Dynamically converging tastes

Bergstrom (1989) describes two individuals, Romeo and Juliet, each taking into account the other's wellbeing when evaluating alternatives. In his model, the wellbeing of Romeo is affected by Romeo's own individual tastes over goods, as well as by Juliet's wellbeing, and the same for Juliet. In contrast to Bergstrom's model the decision maker we have in mind does not take under advisement others' consideration of her or his own individual tastes. Rather, the decision maker's individual tastes are accounted for only directly, through the utility from consuming alone. For example, when buying takeout for dinner with friends, a subjective utilitarian decision maker considers his or her own individualistic preferences, as well as the friends' individualistic preferences, over different types of food. This is as opposed to taking under advisement friends' preferences that the decision maker like the food as well.

The subjective utilitarian model offered in this paper can be studied as part of a dynamic feedback system, that allows for considerations as in Bergstrom's paper. Utilities within the feedback system converge to a steady state, with limiting utilities that are weighted sums of individuals' initial utilities and the others' steady-state utilities. Thus, individual i takes under advisement both her or his original tastes, as well as the others' (eventual) tastes, which themselves depend on i's own tastes. This is explained in more detail in Supplementary Appendix C.

4. Proofs

4.1. Proof of Proposition 1

Assumptions B2-B5 imply that for each fixed social context $G \subseteq I$, $\succsim$ over$\{(p, G) \mid p \in Y\}$ is a weak order, satisfying archimedeanity and independence, therefore it is represented by a vNM utility function. For each $G \subseteq I$ denote the corresponding utility function by u_G, calibrated to as to assign $u_G(r_*) = 0$ and $u_G(r^*) = 1$, for r^* and r_* the consequences which existence is postulated in B1. Therefore for any group G and every $\alpha \in (0, 1)$, $u_G(\alpha r^* + (1-\alpha)r_*) = \alpha$, and by B1 and Independence (B5), for any two groups G and H and every $\alpha \in [0, 1]$, $(\alpha r^* + (1 - \alpha)r_*, G) \sim (\alpha r^* + (1-\alpha)r_*, H)$.

Let m denote the lottery $0.5r^* + 0.5r_*$. Let p, q be lotteries and G and H groups, and consider the alternatives (p, G) and (q, H). Following B1 and Independence (B5), for every $\lambda \in (0, 1)$, $(p, G) \succsim (q, H)$, if and only if, $(\lambda p + (1 - \lambda)m, G) \succsim (\lambda q + (1 - \lambda)m, H)$. By the calibration of u and the definition of m, there exists $0 < \lambda \leq 1$ small enough such that $0 < u_G(\lambda p + (1 - \lambda)m), u_H(\lambda q + (1 - \lambda)m) < 1$. Denote $\lambda p + (1 - \lambda)m$ and $\lambda q + (1 - \lambda)m$ for such a λ by p_m and q_m, respectively.

Suppose that $(p_m, G) \succsim (q_m, H)$. For any $\alpha \in [0, 1]$, Transitivity (B3) implies that if $(q_m, H) \succsim (\alpha r^* + (1-\alpha)r_*, H)$ then since $(\alpha r^* + (1-\alpha)r_*, H) \sim (\alpha r^* + (1-\alpha)r_*, G)$ (following B1 and Independence), it holds that $(p_m, G) \succsim (\alpha r^* + (1 - \alpha)r_*, G)$. Using the vNM utilities u_G and u_H, this is equivalent to concluding that $u_H(q_m) \geq \alpha$ implies $u_G(p_m) \geq \alpha$, for every $\alpha \in [0, 1]$. Therefore, $u_G(p_m) \geq u_H(q_m)$. If, on the other hand, $(q_m, H) \succ (p_m, G)$ then, again by Transitivity, for any $\alpha \in [0, 1]$, $u_G(p_m) \geq \alpha$ implies $u_H(q_m) > \alpha$. Hence $u_H(q_m) > u_G(p_m)$. It follows that $(p_m, G) \succsim (q_m, H)$ if and only if $u_G(p_m) \geq u_H(q_m)$. Since $u_G(p_m) = \lambda u_G(p) + (1 - \lambda)0.5$, and similarly for $u_H(q_m)$, for λ strictly larger than zero, it is established that $(p, G) \succsim (q, H)$, if and only if, $u_G(p) \geq u_H(q)$.

For the uniqueness up to a joint shift and scale, suppose that $\hat{u}_G$, $G \subseteq I$, is another array of vNM utilities representing $\succsim$ in the same manner as the utilities u_G. Since both arrays are of vNM utilities, then for each group G, $\hat{u}_G = \sigma_G u_G + \tau_G$, for $\sigma_G > 0$ and some τ_G. However, for every group G, $\hat{u}_0(r_*) = \sigma_0 u_0(r_*) + \tau_0 = \tau_0 = \hat{u}_G(r_*) = \sigma_G u_G(r_*) + \tau_G$, hence all the shifts τ_G coincide, and, $\hat{u}_0(r^*) = \sigma_0 + \tau = \hat{u}_G(r^*) = \sigma_G + \tau$, hence all the scales σ_G coincide.

The other direction is immediate.

4.2. *Proof of Theorem 1*

4.2.1. *Sufficiency: the representation holds*

We employ the utilities u_G, $G \subseteq I$, calibrated so that $u_G(r^*) = 1$ and $u_G(r_*) = 0$, for r^* and r_* the maximal and minimal consequences which existence is postulated in B1. Denote agent zero's utility alone (i.e., for $G = \emptyset$) by u_0. The proof is conducted in the Euclidean space $\mathbb{R}^X$. Each u_G is given by a vector of real numbers in this space, where this vector's x-th coordinate indicates the utility $u_G(x)$, namely the utility from the prize x in the context of group G. For each of these vectors, the r_*-coordinate is 0 and the r^*-coordinate is 1, according to the chosen normalization. The utility assigned to any lottery p in the context of a group G is $u_G(p) = u_G \cdot p$. Note that following Minimal Disagreement (S4) there are at least two referent individuals, namely, $|I| \geq 2$.

Throughout the proof we use the following terminology:

Definition 1. Two utilities, u and u', are *proportional*, if $u' = cu$ for $c > 0$. They are *antipodes* if $u' = cu$ for $c < 0$.

Consider the negation of Individual Zero's utility, $-u_0$. If we think of the vNM utility $u_{G\cup H}$ as that of a social planner aggregating the vNM utilities u_G and u_H, and $-u_0$, then the Compromise assumption (S1) is simply a Pareto condition, yielding, as in Harsanyi (1955),[4]

$$u_{G\cup H} = \alpha u_G + \beta u_H + \gamma \cdot (-u_0) + \tau \ , \quad \alpha, \beta, \gamma > 0 \ . \tag{3}$$

Employing the normalization with r^* and r_* yields that $\tau = 0$, and $1 + \gamma = \alpha + \beta$.

For every nonempty group $T \subseteq I$ define $v_T = u_T - u_0$. The resulting function over lotteries v_T is a vNM utility function. We abbreviate $v_{\{i\}}$ to v_i, for every referent individual $i \in I$. The following conclusion is implied by what was written thus far.

Conclusion 1.

(1) *For every nonempty group T, $v_T(r_*) = v_T(r^*) = 0$.*
(2) *For every nonempty G and H, v_G and v_H are not antipodes - follows from our assumption of Socially desirable and undesirable lotteries (S3). It therefore also holds that any two positive combinations of such v vectors are not antipodes.*
(3) *Minimal Disagreement (S4) implies that there are $i, j \in I$ such that v_i, v_j are not proportional.*
(4) *For every nonempty, disjoint G and H, $v_{G\cup H} = \alpha v_G + \beta v_H$ for $\alpha, \beta > 0$.*
(5) *By sequential applications of (4), for every nonempty group T, $v_T = \sum_{i\in T} \lambda_i v_i$, $\lambda_i > 0$ for every $i \in T$.*

[4] We thank an anonymous referee for suggesting this step in the proof, significantly simplifying our original proof.

Claim 1. Let $T \subseteq I$ be a group such that there are $i, j \in T$ for which v_i, v_j are not proportional. Then there is a permutation π of the members of T, such that for every $j = 2, \ldots, |T|$, $\sum_{i<j} v_{\pi(i)}$ and $v_{\pi(j)}$ are not proportional.

Proof. The proof is conducted by induction. First let $\pi(1)$ and $\pi(2)$ be the two individuals in T which are assumed not to be proportional (in any order between the two of those). Now suppose that for some $m = 2, \ldots, |T| - 1$ the ordering $\pi = \{\pi(1), \ldots, \pi(m)\}$ of m members of T satisfies that for every $j = 2, \ldots, m$, $\sum_{i<j} v_{\pi(i)}$ and $v_{\pi(j)}$ are not proportional. Let $\pi(m+1)$ be any of the individuals in T which were not ordered yet within $\{\pi(1), \ldots, \pi(m)\}$. If $v_{\pi(m+1)}$ and $\sum_{i<m+1} v_{\pi(i)}$ are not proportional then we are done. Otherwise $\sum_{i<m+1} v_{\pi(i)} = c v_{\pi(m+1)}$ for $c > 0$.

Swap between $\pi(m)$ and $\pi(m+1)$ to generate a new ordering of $m+1$ members of T, $\pi' = \{\pi(1), \ldots, \pi(m-1), \pi(m+1), \pi(m)\}$. If $v_{\pi(m+1)}$ is proportional to $\sum_{i=1}^{m-1} v_{\pi(i)}$, namely, $\sum_{i=1}^{m-1} v_{\pi(i)} = d v_{\pi(m+1)}$ for $d > 0$, then $d v_{\pi(m+1)} + v_{\pi(m)} = c v_{\pi(m+1)}$. Since $v_{\pi(m)}, v_{\pi(m+1)}$ are not antipodes (see (2) in the conclusion above), they must be proportional, resulting that $v_{\pi(m)}$ and $\sum_{i=1}^{m-1} v_{\pi(i)}$ are proportional, a contradiction to the induction assumption. Hence $v_{\pi(m+1)}$ and $\sum_{i=1}^{m-1} v_{\pi(i)}$ are not proportional. It is left to show that $v_{\pi(m)}$ is not proportional to $\sum_{i=1}^{m-1} v_{\pi(i)} + v_{\pi(m+1)}$. Suppose on the contrary that $\sum_{i=1}^{m-1} v_{\pi(i)} + v_{\pi(m+1)} = d v_{\pi(m)}$ for $d > 0$. It thus holds that $\sum_{i=1}^{m-1} v_{\pi(i)} + \frac{1}{c} \sum_{i=1}^{m} v_{\pi(i)} = d v_{\pi(m)}$, that is, $\sum_{i=1}^{m-1} (1 + \frac{1}{c}) v_{\pi(i)} = (d - \frac{1}{c}) v_{\pi(m)}$. It cannot be that $(d - \frac{1}{c}) \leq 0$, on account of (2) of Conclusion 1 and the fact that there is a lottery that is positively evaluated (S3). On the other hand, $(d - \frac{1}{c}) > 0$ contradicts the induction assumption. Hence the new ordering π' satisfies that for every $j = 2, \ldots, m+1$, $\sum_{i<j} v_{\pi'(i)}$ and $v_{\pi'(j)}$ are not proportional. And so by induction there is an ordering π that satisfies the desired property. $\square$

Claim 2. Let G, H be two nonempty groups. If for every p, $v_H(p) = 0 \iff v_G(p) = 0$, then v_G and v_H are proportional.

Proof. Suppose that $\forall p,\ v_H(p) = 0 \iff v_G(p) = 0$. If for some $x \in X$, $v_G(x) = 0$, then also $v_H(x) = 0$. Hence v_G and v_H have the same nonzero coordinates. According to Social desirable and undesirable lotteries (S3), v_G has both positive and negative coordinates. Choose one positive and one negative. Then there is a lottery p over only these two coordinates (prizes) which obtains $v_G(p) = 0$. By our assumption also $v_H(p) = 0$, hence these two coordinates in the two vectors are either proportional or antipodes. The same holds for any other negative coordinate of v_G with the same positive coordinate, and for any other positive coordinate of v_G with the same negative coordinate from the first step. Since v_G and v_H are not antipodes (part (2) of Conclusion 1), they must be proportional. $\square$

Claim 3. Let G, H be two disjoint, nonempty groups and suppose that v_G and v_H are not proportional. Then $v_{G \cup H} = v_G + v_H$.

Proof. We already know that $v_{G \cup H} = \alpha v_G + \beta v_H$ for $\alpha, \beta > 0$. It remains to show that $\alpha = \beta = 1$. Suppose on the contrary that $\alpha \neq 1$. According to Consistent Influence (S2), for every lottery p,

$$(p, H) \succsim p \iff (p, G \cup H) \succsim (p, G) \ ,$$
$$(p, H) \precsim p \iff (p, G \cup H) \precsim (p, G) \ .$$

Translating to the representation and using the v functions,

$$v_H(p) \geq 0 \iff \alpha v_G(p) + \beta v_H(p) \geq v_G(p) \ ,$$
$$v_H(p) \leq 0 \iff \alpha v_G(p) + \beta v_H(p) \leq v_G(p) \ .$$

Consequently, $v_H(p) = 0$ if and only if $\beta v_H(p) = (1-\alpha)v_G(p)$, and under the assumption that $\alpha \neq 1$, $v_H(p) = 0$ if and only if $v_G(p) = 0$. By Claim 2 v_H and v_G are proportional, contradicting the assumption in the claim. Therefore $\alpha = 1$.

In the same manner, suppose on the contrary that $\beta \neq 1$, and repeat the above arguments for the assertion, $(p, G) \succsim p$ if and only if $(p, G \cup H) \succsim (p, H)$, and so on. It is concluded that if v_G and v_H are not proportional, then $v_{G\cup H} = v_G + v_H$. □

Conclusion 2. *Let T be a nonempty group such that there are $i, j \in T$ for which v_i, v_j are not proportional. Then $v_T = \sum_{i\in T} v_i$.*

Proof. By applying Claim 1 and then Claim 3 consecutively. □

Claim 4. Let T be a nonempty group. Then $v_T = \sum_{i\in T} v_i$.

Proof. If there are $i, j \in T$ for which v_i, v_j are not proportional then we are done, according to the conclusion above. Otherwise suppose that all the v_i functions for $i \in T$ are proportional. By (3) of Conclusion 1 there is $j \in I$ such that v_j and v_i for some (hence for every) $i \in I$ are not proportional. Consider $v_{T\cup\{j\}}$. According to Conclusion 2, $v_{T\cup\{j\}} = \sum_{i\in T} v_i + v_j$. On the other hand, since all the v_i functions for $i \in T$ are proportional, then by (5) of Conclusion 1, $v_T = \theta v_i$, $\theta > 0$. Therefore v_j and v_T are not proportional, and according to Claim 3, $v_{T\cup\{j\}} = v_T + v_j$. Hence $v_T = \sum_{i\in T} v_i$. □

Uniqueness of u_0 and the v functions up to a joint shift and scale is implied by the uniqueness result of the proposition. Parts (a), (b), and (c) of the theorem are immediate implications of B1, S3, and S4.

4.2.2. Necessity: the axioms hold

Suppose that $\succsim$ is represented as in (ii) of Theorem 1. Assumptions B1-B5, and S3, immediately follow. For Compromise (S1) suppose that for a lottery p and two disjoint groups G and H, it holds that,

$$u_0(q) \geq u_0(p)$$
$$u_0(p) + \sum_{j\in G} v_j(p) \geq u_0(q) + \sum_{j\in G} v_j(q)$$
$$u_0(p) + \sum_{j\in H} v_j(p) \geq u_0(q) + \sum_{j\in H} v_j(q) \ .$$

It follows that,

$$\sum_{j\in G}(v_j(p) - v_j(q)) \geq u_0(q) - u_0(p) \geq 0 \ , \quad \text{and}$$
$$\sum_{j\in H}(v_j(p) - v_j(q)) \geq u_0(q) - u_0(p) \geq 0 \ .$$

Hence,

$$\sum_{j \in G \cup H} (v_j(p) - v_j(q)) \geq 2(u_0(q) - u_0(p)) \geq u_0(q) - u_0(p) \ ,$$

yielding that $(p, G \cup H) \succsim (q, G \cup H)$, as required by Compromise (S1). If any of the first three inequalities are strong then the resulting inequality is strong as well.

For Consistent influence (S2), let G, H and K be pairwise-disjoint, and suppose that $(p, G) \succsim (p, H)$. According to the representation assumed, it follows that $\sum_{j \in G} v_j(p) \geq \sum_{j \in H} v_j(p)$, yielding that $\sum_{j \in G \cup K} v_j(p) \geq \sum_{j \in H \cup K} v_j(p)$, as the sets are pairwise-disjoint. Adding $u_0(p)$ to both sides of this inequality delivers the required preference.

According to part (c) of the theorem there are $i, j \in I$ such that $v_i \neq c v_j$ for $c > 0$. Since v_i, v_j are vNM utility functions, then once one is not an affine transformation of the other, they represent different preferences over lotteries. It implies that for some lotteries the preferences of i and j are reversed, hence Minimal Disagreement (S4).

Appendix A. Supplementary material

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.jet.2020.105108.

References

Bergstrom, T., 1989. Puzzles: love and spaghetti, the opportunity cost of virtue. J. Econ. Perspect. 3, 165–173.

Bolton, G.E., Ockenfels, A., 2000. ERC: a theory of equity, reciprocity, and competition. Am. Econ. Rev. 90, 166–193.

Charnes, G., Rabin, M., 2002. Understanding social preferences with simple tests. Q. J. Econ. 117, 817–869.

Cooper, D., Kagel, J.H., 2013. Other Regarding Preferences: A Selective Survey of Experimental Results. Handbook of Experimental Economics, vol. 2.

Dillenberger, D., Sadowski, P., 2012. Ashamed to be selfish. Theor. Econ. 7, 99–124.

Dufwenberg, M., Heidhues, P., Kirchsteiger, G., Riedel, F., Sobel, J., 2011. Other-regarding preferences in general equilibrium. Rev. Econ. Stud. 78, 613–639.

Fehr, E., Schimdt, K.M., 1999. A theory of fairness, competition, and cooperation. Q. J. Econ. 114, 817–868.

Fershtman, C., Segal, U., 2018. Preferences and social influence. Am. Econ. J. Microecon. 10 (3), 124–142. https://doi.org/10.1257/mic.20160190.

Harsanyi, J.C., 1955. Cardinal welfare, individualistic ethics, and interpersonal comparisons of utility. J. Polit. Econ. 63 (4), 309–321.

Karni, E., Safra, Z., 2002. Individual sense of justice: a utility representation. Econometrica 70, 263–284.

Maccheroni, F., Marinacci, M., Rustichini, A., 2012. Social decision theory: choosing within and between groups. Rev. Econ. Stud. 79, 1591–1636.

Ok, E.A., Koçkesen, L., 2000. Negatively interdependent preferences. Soc. Choice Welf. 17, 533–558.

Postlewaite, A., 2010. Social norms and preferences. In: Benhabib, J., Bisin, A., Jackson, M.O. (Eds.), Handbook of Social Economics. North Holland, Amsterdam, Netherlands, pp. 31–67.

Segal, U., Sobel, J., 2007. Tit for tat: foundations of preferences for reciprocity in strategic settings. J. Econ. Theory 136, 197–216.

von Neumann, J., Morgenstern, O., 1944. Theory of Games and Economic Behavior, 2nd edition 1947, 1953. Princeton University Press, Princeton NJ.