Subjective Utilitarianism: Decisions in a social context*

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Abstract

Individual decisions are often subjectively affected by other-regarding preferences. We present a model whereby a decision maker has a grand group of significant others. Each sub-group of significant others is a possible *social context*, and the decision maker has (potentially) different preferences in different social contexts. An axiomatic characterization of such preferences is offered. The characterized representation taking a simple *Subjective Utilitarian* form: (a) the decision maker ascribes to each significant other a utility function, representing the decision maker's subjective perception of this other person's tastes, and (b) in any specific social context the decision maker evaluates alternatives by adding together her or his own personal utility and the sum of all group members' utilities as subjectively perceived by the decision maker.

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1 Introduction

It is widely recognized in the economic literature that people may have other-regarding preferences, namely preferences that are not strictly selfish, but also depend on others in their society. People may be inequity averse, care about social welfare, or be sensitive to their social status, to name only a few examples. That preferences and decisions of an individual may be affected by others' payoff was demonstrated in divers experiments (see Cooper and Kagel [4] for a survey). On the theoretical side, various models were designed to describe and explore forms of other-regarding preferences. Some examples include Fehr and Schmidt [8] and Bolton and Ockenfels [2], who offer models in which agents are inequity averse; Charness and Rabin [3] who present experiments and a functional form that incorporates social welfare concerns into agents' preferences; and Dufwenberg et al [7], in which general equilibria is explored based on individual other-regarding preferences that depend on all agents' opportunities.[1]

In the majority of models of individual preferences that exhibit other-regarding features, concern for the welfare of others in society is incorporated into the individual preferences in one of two ways. Some models evaluate the welfare of all others using one utility, either the personal utility related to the modelled individual, or some social, universal utility. The approach in those models is paternalistic in that welfare of others is not judged based on their own tastes. Such models are used to describe, for instance, an individual's concern for fairness, which is measured through this individual's eyes or according to some social norm. Other models in the other-regarding literature evaluate the welfare of others using their actual utilities. These models use considerably more observables than models of the first type. Whereas models of the first type require to observe only the preferences of a single individual, models of the second type assume that the preferences of all other individuals are available as well. These models are employed,

[1] For an extensive discussion of other-regarding models see Postlewaite [13]. An especially nice example for the impact of other-regarding preferences on a standard model can be found in Bergstrom [1].

for example, when equilibria is investigated under an assumption of other-regarding preferences of agents.

This paper offers a third way of incorporating concern for others' welfare into the preferences of an individual. Our model attempts to account for others' tastes, yet does not require to observe more than the preferences of the single individual. Before getting into the specifics of the model, we motivate the discussion with two examples. First, imagine a person buying takeout for dinner with friends. This person may personally prefer Italian to Chinese food, but settle for Chinese food nevertheless, since her or his friends prefer it to Italian. Similarly, consider a person booking a family vacation. She or he may personally rather travel to Paris than to an all-inclusive resort, yet opt for the resort if her or his children like the resort better.

In both examples, an individual choosing one alternative over another takes into account the tastes of others that will be affected by this decision. The choice itself (e.g., the purchase of takeout food, the booking of a vacation) is made by the individual alone, however consumption of the chosen commodities is shared with a group of significant others (e.g., the decision maker's friends, the decision maker's family). We claim that in cases of this sort, a decision is affected by the group with which consumption is made just as it is affected by the purchased good itself (e.g., the group of friends attending dinner affects the decision of which takeout food to buy, family members affect the decision of which vacation to book). Thus we may see an individual preferring one alternative over another on one occasion, but exhibiting the reversed preference on another occasion, where this reversal is not a consequence of indifference or inconsistency, but of considering consumption of the same goods with different groups of people.

The model proposed in this paper is designed to address decisions of the sort described above. Within the model, the preferences of a single individual are considered. Groups of others with which this individual consumes are made explicit, and taken into account in the individual's preferences. This allows us to accommodate a decision maker who is affected by such groups, possibly exhibiting reversals of preferences over goods which result from considering different reference groups.

Formally, one of the primitives assumed is a grand group of significant others, which is meant to include all those people who are important to the decision maker, and may affect his or her decisions. Consumption is possible with any sub-group of this grand group, and each of these sub-groups is called a *social context*, or a *group context*. Alternatives take social contexts into account, in that the individual preferences considered are over pairs of a lottery and a social context, namely a sub-group of significant others with which the lottery will be consumed. The setup is therefore a classic von-Neumann and Morgenstern one (vNM; [14]), with one simple addition of groups of referent others. With this choice of setup it is possible to accommodate, for instance, an individual whose strict preference is to spend a vacation with her or his spouse in Paris rather than in an all-inclusive resort, but exhibits the reversed preference, for the resort over Paris, when considering a vacation with the entire family. Moreover, this setup allows us to address dual questions, such as, whether an individual prefers to spend a vacation in Paris with her or his spouse alone, to spending it with her or his spouse and kids. In fact, these types of questions, in which one component in the compared alternatives is kept constant, are the only types of questions posed within the model. Comparisons which involve both different lotteries and different groups are not required to be determined. This is since we believe such comparisons are more complex and hence difficult to determine, and furthermore not readily observed, even in cases where they are determined.

A possible critique at this point would be, that a vacation with one's spouse in Paris, and a vacation with one's entire family in Paris, are simply two different alternatives. They can be modelled as such in an abstract manner, without imposing a structure which contains a social context. Our reply is that an explicit modelling of a social context allows us not only to distinguish between those two alternatives, but moreover to ask if and how the decision maker's evaluation of those alternatives differs as a function of the social context involved. And indeed our aim is to identify a systematic dependency of preferences on a social context. To compare, vacation today and vacation tomorrow are also two different alternatives. However, specifying time as part of an alternative's description facilitates a characterization of time discounted preferences.

Having a setup which supports an investigation of preferences depending on a social context, the question still remains, what kind of dependency does the model depict? We suggested above that people take into account others' tastes when forming a decision. On the other hand, we insisted on a model that relies on observing the preferences of a single individual. But without observing the preferences of others, it is impossible to extract their tastes, and so how can their tastes be taken into account?

To answer this question, we return to the examples above. These examples describe a person buying Chinese rather than Italian food on the premise that her or his friends prefer Chinese, and another person booking a family vacation in an all-inclusive resort based on the view that her or his kids prefer it to a vacation in Paris. The decisions in both cases are made by one person alone. Relevant others do not affect these decisions directly, but rather indirectly through the decision maker's perception of their tastes. This perception is by its nature purely subjective, and may or may not be aligned with the others' true tastes. Nevertheless, this purely subjective perception of tastes, as well as the decision maker's inclination to be considerate of others' preferences, are what drives the decision maker's other-regarding behavior. The representation offered here thus depends not on others' true utilities (which are not observed), but on the way those utilities are perceived by the decision maker.

Specifically, the decision maker in our model takes others' tastes into account in a *subjective utilitarian* manner. That is to say, the evaluation described is as if the decision maker, on top of entertaining a personal vNM utility function, subjectively ascribes a vNM utility function to each significant other. A lottery in a group context is then evaluated through the utilitarian sum of the decision maker's personal vNM utility from this lottery, and the vNM utilities from this lottery of all group members, as subjectively perceived by the decision maker. The work in the paper is axiomatic, behaviorally characterizing subjective utilitarianism. The main assumptions in the identification of a subjective utilitarian individual require that the effect of considering social contexts be consistent, both when considering groups with shared members, and in the preference reversals that are inflicted. Preference reversals are interpreted as representing compromises made by the decision maker in favor of significant others.

Formally, a pair (p, G) of a lottery p in the context of a group G is evaluated within the representation by,

$$V(p, G) = u_0(p) + \sum_{j \in G} v_j(p), \tag{1}$$

where u_0 is the personal vNM utility of the decision maker, and v_j is the vNM utility subjectively ascribed to individual j by the decision maker.

Central to the representation is the fact that the decision maker attributes to each referent other one vNM utility once and for all, and this same utility is used whenever a lottery is evaluated in the context of a group containing this referent other. We interpret this utility as reflecting the tastes of this referent individual, as these are perceived by the decision maker. The representation theorem furthermore delivers that utilities ascribed to others are calibrated relative to the decision maker's own utility. That is to say, others' utilities are unique once the decision maker's personal utility is fixed. Therefore, embedded into the vNM utilities ascribed to others, is the extent to which the decision maker wishes to comply with the tastes they represent.

Our interpretation of the model is of an individual making decisions in consideration of others' welfare, as this welfare is perceived by her or him. An alternative, paternalistic interpretation of the model is as depicting a decision maker who bases decisions on her or his own judgement of what is better for significant others. For instance, the decision maker may form preferences based on the contention that it is better for her or his children to listen to a classical concert than to watch an action movie, even if the children's preferences are opposite. Another alternative interpretation of the model is as describing a decision maker who is driven by spite. In that case the functions v_j

represent disutilities of referent others rather than their utilities.

The representation suggested here constitutes a subjective version of Harsanyi's utilitarianism [9]. Here as well the tastes of all individuals in society are taken into account in an additive manner. However, while the choice of weights in Harsanyi's model poses difficult ethical questions regarding interpersonal utility comparisons in a society, in the model discussed here the preferences at issue are of a single individual, therefore no ethical dilemma arises. The calibration of others' utilities within the subjective utilitarian model simply reflects the degree to which the decision maker wishes to be considerate of others' tastes.

We conclude the introduction with a discussion of the most related literature. Most closely related to our work are axiomatic papers which characterize other-regarding representations of individual preferences. In all these models that we are aware of, however, welfare of others is evaluated either based on the decision maker's personal tastes, or according to some social measure, which is the same for all. Namely, in contrast to our model, welfare of others in those models is not evaluated in a manner that accounts for others' own tastes. Accordingly, these models cannot describe decision makers who are considerate of other people's preferences (even if subjectively), but address issues such as status or fairness concerns.

Among the axiomatic models of individual other-regarding preferences is Maccheroni, Marinacci, and Rustichini [11]. The model contains characterizations of individual preferences over allocations of general acts to sub-groups of agents, portraying a decision maker who is sensitive to her or his social status. The decision maker compares her or his allocated acts to others' allocated acts through a personal utility, or by applying one other utility, the same for all agents, interpreted as a social value function. Another axiomatic paper involving an individual with social status concerns is Ok and Koçkesen [12], describing a preference over income distributions which representation reflects the individual's wish to occupy a higher status than others in society.

A different motivation for other-regarding preferences is expressed in Karni and Safra

[10]. These authors describe an individual choice behavior which is driven by ethical motives in addition to the more traditional purely-selfish motives. The paper offers a representation which involves one function representing the purely selfish preferences of a decision maker, and another one that represents his or her moral preferences. Lastly we mention Dillenberger and Sadowski [6], characterizing an individual decision maker who chooses between menus of payoff allocations to himself or herself and another individual, motivated by a tradeoff between self interest and a wish not to appear selfish.

The paper is organized as follows. Section 2 describes the model, the axioms and the main results. Section 3 contains comments and some extensions. All the proofs appear in Section 4.

2 The Model and Main Results

2.1 Setup

Suppose a finite set of prizes X, and a set Y of lotteries over X, namely probability distributions over X with a finite support. We are interested in the preferences of an individual when the individual operates in a group context. The individual whose preferences we examine is called 'Individual Zero', and the grand set of referent individuals for Individual Zero is denoted $I = \{1, \ldots, N\}$ with $N > 2$, each $i = 1, \ldots, N$ being a 'significant other' for Individual Zero. A *social context*, or a *group context* is a subset $G \subseteq I$. Consuming lottery p when Individual Zero is with a group G, which we refer to as consuming p in a context G, is denoted (p, G). Consuming p when Individual Zero is alone is written simply as p (shortening $(p, \emptyset)$). Individual Zero's preferences, denoted $\succsim$, are over such pairs of lotteries and contexts, but we only require that preferences be expressed once one of these components is constant. Thus, the modelled individual performs either comparisons of the form $(p, G) \succsim (q, G)$, or of the form $(p, G) \succsim (p, H)$. The former is interpreted as stating that when with group G, the individual finds lottery p as at least as preferred as lottery q. The latter is interpreted as stating that the individual weakly prefers to consume lottery p with group G than to consume it with group H. Formally, $\succsim \subseteq \mathcal{R}$, for

$$\mathcal{R} = \left(\bigcup_{G \subseteq I} \bigcup_{p,q \in Y} ((p,G),(q,G)) \right) \bigcup \left(\bigcup_{p \in Y} \bigcup_{G,H \subseteq I} ((p,G),(p,H)) \right) .$$

The symmetric and asymmetric components of $\succsim$ are respectively denoted $\sim$ and $\succ$.

A structural assumption is imposed at the outset, whereby it is postulated that there are purely individualistic better and worse prizes in X. These are prizes which consumption is unaffected by a context, and are equally preferred when consumed with or without a group of others. For instance, Individual Zero may prefer red apples to green ones, and eating either of these apples with or without a group is not likely to make a difference.

C0. Individualistic ranking.
There are $x^*, x_* \in X$ such that $x^* \succ x_*$, and for any group G, $(x^*, G) \sim x^*$ and $x_* \sim (x_*, G)$.

2.2 Basic representation

Our first step in characterizing Individual Zero as a subjective utilitarian is to identify when the preferences of this individual, both within and across contexts, are represented by a family of vNM utilities. The first assumption in this basic characterization directly asserts that given a fixed context, the preferences of Individual Zero over lotteries in this fixed context satisfy the axioms of von-Neumann and Morgenstern (vNM; see [14]). Thus, within any fixed context, Individual Zero's preferences admit a representation by a vNM utility function.

C1. vNM Preferences.
Let $G \subseteq I$ be a group. Then $\succsim$ over $\{(p,G) \mid p \in Y\}$ satisfies the vNM [14] axioms (Weak order, Independence, Archimedeanity).

Note that G can also be the empty group, hence the preferences of Individual Zero alone also admit a vNM representation.

The above axiom trivially implies the use of vNM utilities in comparing lotteries within the same group context. The next three axioms pertain to the preferences of Individual Zero across contexts. Firstly, it is assumed that given a fixed lottery, Individual Zero's preferences over groups with which to consume this lottery are complete and transitive.

C2. Weak order over contexts.
Let p be a lottery and G, H and K groups. Then either $(p,G) \succsim (p,H)$ or $(p,H) \succsim (p,G)$, and if $(p,G) \succsim (p,H)$ and $(p,H) \succsim (p,K)$ then $(p,G) \succsim (p,K)$.

Next, a form of independence of the preference across contexts is presumed: given separate preferences to consume each of two lotteries with one group over another, the same holds for any mixture of these two lotteries.

C3. Independence over contexts.
Let p, q be lotteries and G, H groups. If $(p,G) \succsim (p,H)$ and $(q,G) \succsim (q,H)$ then for every $\alpha \in (0,1)$, $(\alpha p + (1-\alpha)q, G) \succsim (\alpha p + (1-\alpha)q, H)$. Moreover, the conclusion holds strictly whenever any of the two antecedents holds strictly.

Lastly transitivity is strengthened, to prevent the relation over lottery-group pairs from generating cycles.

C4. Transitivity across contexts.

Let p, q be lotteries and G, H groups. If $(p, G) \succsim (p, H)$, $(p, H) \succsim (q, H)$, and $(q, H) \succsim (q, G)$, then $(p, G) \succsim (q, G)$. If any one of the first three relationships is strict, then the relationship in the conclusion is strict as well.

In the proposition that follows, it is stated that under the structural assumption of individualistic better and worse prizes (**C0**), axioms **C1-C4** are equivalent to a vNM representation of preferences both within and across contexts. Within each context the result follows trivially from the assumption of vNM preferences (**C1**). The fact that those same utilities may be used to compare the consumption of a lottery across contexts is implied by the other three axioms (**C2-C4**).

Proposition 1. *Let $\succsim \subseteq \mathcal{R}$ and suppose that **C0** holds. Then **C1-C4** are satisfied, if and only if, there exist vNM utility functions u_G, $G \subseteq I$, such that for any two lotteries p and q and groups G and H,*

$$\begin{aligned} (p, G) \succsim (q, G) \quad &\Longleftrightarrow \quad u_G(p) \geq u_G(q) \\ (p, G) \succsim (p, H) \quad &\Longleftrightarrow \quad u_G(p) \geq u_H(p) \end{aligned}$$

Furthermore, these utilities are unique up to joint shift and scale.[2]

2.3 Subjective utilitarianism

Following the previous proposition, Individual Zero applies a family of vNM utilities, one per group of significant others, to compare lotteries both within and across contexts. To further obtain that these utilities take a subjective utilitarian form, three additional axioms are imposed. The first, termed *Compromise*, addresses reversals of personal preferences when Individual Zero is joined by groups of referents. Compromise states

[2]That is to say, if $\hat{u}_G, G \subseteq I$, is any other array of utilities representing $\succsim$ in the same manner, then there are $\sigma > 0, \tau$ such that, $\hat{u}_G = \sigma u_G + \tau$, for every G. For $G = \emptyset$, the utilities u_G and $\hat{u}_G$ are the utility functions of Individual Zero alone.

that if a personal preference is reversed in the context of group G as well as in the context of group H, then it will also be reversed when consumption is made with members of both these groups together. The axiom supports the interpretation that reversal of personal preferences is the result of compromising with others whose preferences are opposite. For if the opposite preferences of members of G are strong enough to make Individual Zero reverse his/her personal preference, and the same holds separately for H, then the preferences of members of G and H together give even more of a reason to compromise. This condition eliminates the possibility of cross-effects when referent individuals are joined together.

C5. Compromise.
Let p, q be lotteries and G, H disjoint groups. If $q \succsim p$, $(p, G) \succsim (q, G)$, and $(p, H) \succsim (q, H)$, then $(p, G \cup H) \succsim (q, G \cup H)$.

Another form of consistency is imposed on the individual preference, whereby a decision with which of two groups to consume a lottery p is determined only by those individuals that belong only to one of these groups. For an example suppose that Individual Zero prefers to go to a classical concert with one friend rather than with another. Then this axiom postulates that Individual Zero would also prefer going to the concert with her or his spouse and the first friend, rather than going with the spouse and the second friend. This is since the basic preference is presumably because the first friend likes classical music better than the second, and as the spouse will go in both alternatives, regardless of his or her preference for classical music, the decision will still be determined by the friends' liking of classical music. As in the previous assumption, this condition rules out cross effects between different individuals.

C6. Consistent Influence.
For every three pairwise-disjoint groups, G, H and K, $(p, G) \succsim (p, H)$ if and only if

$(p, G \cup K) \succsim (p, H \cup K)$.

Lastly, we impose a Richness condition, which allows us to pinpoint uniquely the subjective vNM utilities that the individual ascribes to each of the referent individuals.

C7. Richness.

For any three nonempty, pairwise-disjoint groups G, H and K, there are a lottery p such that $(p, G) \succ (p, H) \succ (p, K) \succ p$, and a lottery q such that $(q, G) \succ (q, H) \succ q \succ (q, K)$.

In order for the assumptions above to be necessary and sufficient for a subjective utilitarian representation, a condition on the resulting utilities is required, that implies the Richness assumption (**C7**). This is the condition formulated next.

Definition 1. A collection of utilities $(u_G)_{G \subseteq I}$ is *diversified* if for every choice of pairwise-disjoint groups G, H, and K, every convex combination of $u_G - u_H$, $u_H - u_K$, and $u_K - u_\emptyset$, as well as any convex combination of $u_G - u_H$, $u_H - u_\emptyset$, and $u_\emptyset - u_K$, yields at least one strictly positive coordinate.

Our main theorem states that under the structural assumption **C0**, axioms **C1** through **C7** are equivalent to a subjective utilitarian representation: Individual Zero ascribes to each referent individual a subjective vNM utility function, and evaluates a lottery in a group context by adding to his or her own personal utility from this lottery the sum of the subjective utilities from this lottery, of all group members. In other words, Individual Zero's evaluation of lotteries in the context of a group is utilitarian, comprised of the individual's personal utility and the subjective utilities of all group members. Importantly, the utility of each referent individual is the same in all groups to which this individual belongs.

Theorem 1. *Let* $\succsim \subseteq \mathcal{R}$ *be a binary relation and suppose that* ***C0*** *holds. Then the following two statements are equivalent:*

(i) Assumptions ***C1-C7*** *hold.*

(ii) There exist vNM utilities, $u_0, v_1, \dots, v_N$*, such that for any lotteries* p *and* q *and groups* G *and* H*,*

$$\begin{aligned}(p, G) \succsim (q, G) \quad &\Longleftrightarrow \quad u_0(p) + \sum_{j \in G} v_j(p) \geq u_0(q) + \sum_{j \in G} v_j(q) \\ (p, G) \succsim (p, H) \quad &\Longleftrightarrow \quad \sum_{j \in G} v_j(p) \geq \sum_{j \in H} v_j(p)\end{aligned}$$

Furthermore, $u_0, v_1, \dots, v_N$ *are unique up to a joint scale and a shift of* u_0*, and the resulting utilities,* $u_G = u_0 + \sum_{j \in G} v_j$*, for* $G \subseteq I$*, are diversified.*[3]

The proof appears in Section 4.

The representation together with **C0** implies that for every referent other j, $v_j(x_*) = v_j(x^*) = 0$. The outcomes x_* and x^* hence serve as a threshold for the decision maker, being outcomes towards which others are indifferent, à-la the decision maker. It follows that individual j's evaluation of a lottery p, as this is perceived by the decision maker, is positive, if and only if, $v_j(p) > v_j(x_*)$. Consequently, for a fixed lottery p, the decision maker will be better off if a referent other j with $v_j(p) > v_j(x_*)$ joins the social context in which p is consumed, and worse off if strict inequality in the other direction holds for an individual joining the group.

When there are less than three additional individuals, a less specific representation can be derived.

Remark 1. When $I = \{1\}$ the result follows trivially. When $I = \{1, 2\}$ we can prove that assumptions **C1-C6**, together with a richness condition that postulates the

[3]The utility $u_\emptyset$ is simply u_0, the personal utility of the individual under consideration.

existence of a lottery p for which $(p, \{1\}) \succ (p, \{2\}) \succ p$, are equivalent to the existence of vNM utilities, u_0, v_1, and v_2, unique up to a joint scale and a shift of u_0, such that for any lottery p the alternative (p, G) is evaluated by: $u_0(p)$, for $G = \emptyset$; $u_0(p) + v_i(p)$, for $G = \{i\}$, i=1,2; $u_0(p) + \lambda_1 v_1(p) + \lambda_2 v_2(p)$, for $G = \{1, 2\}$, where $\lambda_1, \lambda_2 \geq 0$ and not both are zero.

3 Comments

3.1 Measuring others' influence

In our model, a preference of a decision maker over lotteries may be reversed when the decision maker consumes those lotteries with others. Given such influence of other individuals, a natural comparative question that arises pertains to the degree of influence each of these others has. To answer this question, the notion of 'influence' needs to be precisely defined. Generally speaking, we consider 'influence' to be a case when a significant other causes a reversal of the decision maker's choice. Thus, for a fixed pair of lotteries p and q, a referent other influences the decision maker whenever the decision maker chooses q over p without that referent other, yet reverses this preference to p over q with that same referent. This kind of reversal can occur when a referent other joins only the decision maker, or when the preference of the decision maker in the context of a group is reversed once that referent other joins the group. Instances of influence across different pairs of lotteries are combined by taking the average over all possible pairs (p, q).

To capture reversal of preferences given fixed lotteries p and q, and to facilitate the measurement of others' influence, a simple game is defined:

$$w_{p,q}(S) = \begin{cases} 1 & (p, S) \succsim (q, S) \\ 0 & \text{otherwise} \end{cases}$$

for every group $S \subseteq I$.

Let $v_1, \ldots, v_N$ be the subjectively ascribed utilities from Theorem 1. A referent other $i \in I$ may swing a coalition from being a losing coalition in $w_{p,q}$ to being a winning coalition, if $v_i(p) > v_i(q)$, and may swing a coalition from winning to losing if a strict inequality in the other direction holds. The Banzhaf value of each player $i \in I$ in the above game may be computed as follows:

$$\beta_i(w_{p,q}) = \frac{1}{2^{|N|-1}} \sum_{S \subset I \setminus \{i\}} \left[w_{p,q}(S \cup \{i\}) - w_{p,q}(S) \right] .$$

Since a player i with $v_i(q) > v_i(p)$ can swing coalitions from being winning to being losing, the Banzhaf value of players may be negative. However, we are interested in an influence of significant others, no matter in which direction of preference. Moreover, a player who gains a negative Banzhaf value in $w_{p,q}$ will gain a positive Banzhaf value in the symmetric game $w_{q,p}$. In order to measure influence per se, without indicating in which direction of preference is takes place, and since the measure we aim at will eventually average over all pairs of lotteries p and q, we define:

$$B_i(p,q) = \max\left(\beta_i(w_{p,q}), 0\right) .$$

Altogether, the influence of referent individual i on the decision maker is the average,

$$B_i = \int_{(p,q)} B_i(p,q) d\lambda,$$

where λ is the Lebesgue measure over Y^2. Note that by taking this average, swings are counted whenever they occur (either for p over q or the other way around).

We lastly show that the influence measure defined is sub-additive in the following sense: consider a decision problem derived from the original one by amalgamating two referent individuals into a single individual, whose utility is the sum of the two referents' utilities. Then the influence of the amalgamated individual on the decision maker can never be more than the sum of influences of the two separate referent individuals.

Formally, let $i, j \in I, i \neq j$. For a pair of lotteries p and q, define the game $\bar{w}_{p,q}$ in which i and j are amalgamated into one player $\bar{ij}$ by,

$$\bar{w}_{p,q}(S) = \begin{cases} w_{p,q}(S) & \bar{ij} \notin S \\ w_{p,q}(S \cup \{i,j\} \setminus \{\bar{ij}\}) & \bar{ij} \in S \end{cases}$$

for every $S \subseteq I \setminus \{i,j\} \cup \{\bar{ij}\}$. Denote $\bar{B}_{\bar{ij}}(p,q) = \max(\beta_i(\bar{w}_{p,q}), 0)$, and $\bar{B}_{\bar{ij}}$ the corresponding average over pairs of lotteries p and q. Then,

Proposition 2. $\bar{B}_{\bar{ij}} \leq B_i + B_j$.

A subjective utilitarian decision maker is considerate of the welfare of each significant other individually, in that the personal tastes of each such other are always taken under advisement, and to the same extent, regardless of the group to which this referent other joins. This is conveyed through our axioms of Compromise (**C5**) and Consistent Influence (**C6**), and is translated to additivity in the subjective utilitarian functional. As a result, the only non-additive effect on influence of uniting two individuals together can be when their tastes are opposite, and so their individual influences cancel out when they are considered together. This is the effect described in the proposition.

Proposition 2 states that the influence of a couple of individuals who decide to marry reduces compared to their total influence when they are separated.

3.2 When do subjective utilities equal true utilities?

The model presented in this paper focuses on the preferences of a single individual, asking how these change as a function of the group with which the individual consumes. The expression of others' tastes in the representation is purely subjective, namely, it represents the decision maker's perception of others' tastes, rather than their actual tastes. Put differently, observed decisions of an individual may be based on misperceived preferences of others. Nonetheless, in cases where others' actual preferences may be observed, it is interesting to understand when perceived and actual preferences coincide. Perhaps not surprisingly, such coincidence is essentially a result of a Pareto-type condition.

Denote by $\succsim^i$ the actual preferences of individual i, with symmetric and asymmetric components $\sim^i$ and $\succ^i$, respectively. Suppose that those as well are vNM preferences. On top of this, two assumptions are made at the outset:

(a) For x^* and x_* the individualistic better and worse outcomes from assumption **C0**, $x^* \sim^i x_*$.

(b) There are $x^0, x_0 \in X$ such that both $x^0 \succ x_0$ and $x^0 \succ^i x_0$.

That is to say, individual i is indeed indifferent between the two individualistic outcomes of Individual Zero that appear in the structural assumption **C0**, and individual i and Individual Zero agree on some strict ranking of outcomes. Under these two assumptions, and supposing that the conditions of Theorem 1 hold, the utility of i as subjectively perceived by Individual Zero coincides with i's true utility, if and only if, whenever individual i and Individual Zero personally agree on a ranking of lotteries, this same ranking holds for the preferences of Individual Zero in the company of i. This is stated in the following proposition.

Proposition 3. *Let $\succsim^i$ be a binary relation over Y, represented by a vNM utility function. Let $\succsim \subseteq \mathcal{R}$ be a binary relation that satisfies **C0** and (ii) of Theorem 1. Denote by v_i the subjective utility ascribed by Individual Zero to referent individual i. Suppose that assumptions (a) and (b) above are satisfied. Then v_i represents $\succsim^i$, if and only if, for every two outcomes x and y, if $x \succ y$ and $x \succ^i y$, then $(x, \{i\}) \succ (y, \{i\})$.*

4 Proofs

4.1 Proof of Proposition 1

According to **C1**, for every group $G \subseteq I$, $\succsim$ over $\{(p, G) \mid p \in Y\}$ is represented by a vNM utility function. For each $G \subseteq I$ denote the corresponding utility function by u_G, calibrated to as to assign $u_G(x_*) = 0$ and $u_G(x^*) = 1$, for x^* and x_* the consequences which existence is postulated in C0. Therefore for any group G and every $\alpha \in (0,1)$, $u_G(\alpha x^* + (1-\alpha)x_*) = \alpha$, and by C0, C2 and C3, for any two groups G and H and every $\alpha \in [0,1]$, $(\alpha x^* + (1-\alpha)x_*, G) \sim (\alpha x^* + (1-\alpha)x_*, H)$.

Let m denote the lottery $0.5x^* + 0.5x_*$. Let p be a lottery and G and H groups, and suppose that $(p, G) \succsim (p, H)$. following **C1**, **C2** and **C3**, for every $\lambda \in (0,1)$, $(p, G) \succsim (p, H)$, if and only if, $(\lambda p + (1-\lambda)m, G) \succsim (\lambda p + (1-\lambda)m, H)$, and there exists a large enough such λ such that $0 < u_G(\lambda p + (1-\lambda)m), u_H(\lambda p + (1-\lambda)m) < 1$. Denote $\lambda p + (1-\lambda)m$ for such a large enough λ by p_m.

For any $\alpha \in [0,1]$, **C4** implies that if $(p_m, H) \succsim (\alpha x^* + (1-\alpha)x_*, H)$ then $(p_m, G) \succsim (\alpha x^* + (1-\alpha)x_*, G)$. Using the vNM utilities u_G and u_H, this is equivalent to concluding that $u_G(p_m) \geq \alpha$ whenever $u_H(p_m) \geq \alpha$, for every $\alpha \in [0,1]$. If, on the other hand, $(p_m, H) \succ (p_m, G)$ then, again by C4, for every $\alpha \in [0,1]$, $u_G(p_m) \geq \alpha$ implies $u_H(p_m) > \alpha$. It follows that $(p_m, G) \succsim (p_m, H)$ if and only if $u_G(p_m) \geq u_H(p_m)$. Since $u_G(p_m) = \lambda u_G(p) + (1-\lambda)0.5$, and similarly for $u_H(p_m)$, it is established that $(p, G) \succsim (p, H)$, if and only if, $u_G(p) \geq u_H(p)$.

For the uniqueness up to a joint shift and scale, suppose that $\hat{u}_G, G \subseteq I$, is another array of vNM utilities representing $\succsim$ in the same manner as the utilities u_G. Since both arrays are of vNM utilities, then for each group G, $\hat{u}_G = \sigma_G u_G + \tau_G$, for $\sigma_G > 0$ and some τ_G. However, for every group G, $\hat{u}_0(x_*) = \sigma_0 u_0(x_*) + \tau_0 = \tau_0 = \hat{u}_G(x_*) = \sigma_G u_G(x_*) + \tau_G$, hence all the shifts τ_G coincide, and, $\hat{u}_0(x^*) = \sigma_0 + \tau = \hat{u}_G(x^*) = \sigma_G + \tau$, hence all the scales σ_G coincide.

The other direction is immediate.

4.2 Proof of Theorem 1

4.2.1 Sufficiency: the representation holds

We employ the utilities u_G, $G \subseteq I$, calibrated so that $u_G(x^*) = 1$ and $u_G(x_*) = 0$, for x^* and x_* the maximal and minimal consequences which existence is postulated in C0. Denote agent zero's utility on his/her own (i.e., for $G = \emptyset$) by u_0. The proof is conducted in the Euclidean space $X^{\mathbb{R}}$. Each u_G is given by a vector of real numbers in this space, where this vector's x-th coordinate indicates the utility $u_G(x)$, namely the utility from the prize x in the context of group G. For each of these vectors, the x_*-coordinate is 0 and the x^*-coordinate is 1, according to the chosen normalization. The utility assigned to any lottery p in the context of a group G is $u_G(p) = u_G \cdot p$.

Let G and H be two nonempty, disjoint groups. According to Richness there is a lottery p such that $(p, G) \succ (p, H) \succ p$, namely $u_G(p) > u_H(p) > u_0(p)$. Similarly to the proof of Proposition 1, let $m = 0.5x^* + 0.5x_*$, and $\lambda \in (0, 1)$ be such that $u_G(p_m), u_H(p_m)$, and $u_0(p_m)$ are all between zero and one, for $p_m = \lambda p + (1 - \lambda)m$. C3 yields that $u_G(p_m) > u_H(p_m) > u_0(p_m)$. According to C0 and C3 and the chosen normalization for the utilities under consideration, for every $\alpha \in (0, 1)$, $u_G(\alpha x^* + (1 - \alpha)x_*) = u_H(\alpha x^* + (1-\alpha)x_*) = u_0(\alpha x^* + (1-\alpha)x_*) = \alpha$. Hence there exists an $\alpha : 1-\alpha$ mixture of x^* and x_* such that, $u_G(p_m) > u_H(p_m) > \alpha = u_G(\alpha x^* + (1 - \alpha)x_*) = u_H(\alpha x^* + (1 - \alpha)x_*) = u_0(\alpha x^* + (1 - \alpha)x_*) > u_0(p_m)$. In other words, there exist two lotteries that u_G and u_H rank in the same manner, while u_0 ranks them differently. Hence u_0 is not a convex combination of u_G and u_H.

Consider the sets $conv(u_0, u_{G\cup H})$ and $conv(u_G, u_H)$. It is proved by negation that the intersection of these two sets cannot be empty. Suppose on the contrary that it is empty. Then by a standard separation theorem, there exist a separating linear functional over $X^{\mathbb{R}}$, denote it a, and a constant c, such that: $a \cdot \varphi \geq c > a \cdot \psi$ for every $\varphi \in [u_G, u_H]$ and every $\psi \in [u_0, u_{G\cup H}]$, thus specifically for $\varphi = u_G, u_H$ and $\psi = u_0, u_{G\cup H}$. Suppose w.l.o.g that $c \geq 0$ (otherwise the inequalities to follow may be reversed, and an analogous proof, with reversed rankings of lotteries, yields the desired result). Separating a into its positive and negative parts, a^+ and a^- respectively, yields the inequalities,

$$\begin{aligned} c + a^- \cdot u_0 &> a^+ \cdot u_0 \\ a^+ \cdot u_G &\geq c + a^- \cdot u_G \\ a^+ \cdot u_H &\geq c + a^- \cdot u_H \\ c + a^- \cdot u_{G\cup H} &> a^+ \cdot u_{G\cup H} \end{aligned}$$

Let $M > 0$ be large enough so that $\sum_x a_x^+ \leq M, \sum_x a_x^- \leq M$, and $c \leq M$, and multiply both sides of all the inequalities by $\frac{1}{2M}$. For all the involved utilities, namely for $u = u_0, u_G, u_H, u_{G\cup H}$, the fact that $u(x_*) = 0$ and $u(x^*) = 1$ and the choice of M result:

$$\begin{aligned} \frac{c}{M} &= u(b), \quad b = (x^*, c/M; x_*, 1 - c/M) \\ \frac{a^-}{M} \cdot u &= u(q), \quad \text{for } q \text{ s.t. } q(x) = \frac{a_x^-}{M} \text{ for } x \neq x_* \text{ and } q(x_*) = 1 - \frac{\sum_{x\neq x_*} a_x^-}{M} \\ \frac{a^+}{M} \cdot u &= u(p), \quad \text{for } p \text{ s.t. } p(x) = \frac{a_x^+}{M} \text{ for } x \neq x_* \text{ and } p(x_*) = 1 - \frac{\sum_{x\neq x_*} a_x^+}{M} \end{aligned}$$

Therefore, the inequalities above state (with δ_{x_*} being the lottery yielding x_* with probability 1),

$$\begin{aligned} u_0(\frac{1}{2}b + \frac{1}{2}q) &> u_0(\frac{1}{2}p + \frac{1}{2}\delta_{x_*}) \\ u_G(\frac{1}{2}p + \frac{1}{2}\delta_{x_*}) &\geq u_G(\frac{1}{2}b + \frac{1}{2}q) \\ u_H(\frac{1}{2}p + \frac{1}{2}\delta_{x_*}) &\geq u_H(\frac{1}{2}b + \frac{1}{2}q) \\ u_{G\cup H}(\frac{1}{2}b + \frac{1}{2}q) &> u_{G\cup H}(\frac{1}{2}p + \frac{1}{2}\delta_{x_*}) \ , \end{aligned}$$

generating a contradiction of Compromise (**C5**).

The above implies that the intersection of $conv(u_0, u_{G\cup H})$ and $conv(u_G, u_H)$ cannot be empty. That is, there exist $\alpha \in (0,1]$ and $\theta \in [0,1]$ such that $\alpha u_{G\cup H} + (1-\alpha)u_0 = \theta u_G + (1-\theta)u_H$, where α is strictly positive following the above proof that u_0 is not in itself a convex combination of u_G and u_H.

For every nonempty group T define $v_T = u_T - u_0$. The resulting function over lotteries v_T is a vNM utility function, which for every T satisfies $v_T(x_*) = 0$. Using these utility functions, the above conclusion may be written as, $v_{G\cup H} = \frac{1}{\alpha}\left(\theta v_G + (1-\theta)v_H\right)$, for nonempty disjoint G and H, for some α and θ as above. Translating Consistent Influence (C6) to the v-utilities therefore delivers that for any three nonempty, pairwise-disjoint groups G, H, and K there are $\alpha, \beta \in (0,1]$ and $\theta, \lambda \in [0,1]$, such that,

$$\begin{aligned}
&v_G(p) - v_H(p) \geq 0 \iff \\
&\frac{1}{\alpha}\left(\theta v_G(p) + (1-\theta)v_K(p)\right) \geq \frac{1}{\beta}\left(\lambda v_H(p) + (1-\lambda)v_K(p)\right) \iff \\
&\frac{\theta}{\alpha}v_G(p) - \frac{\lambda}{\beta}v_H(p) \geq \left(\frac{1-\lambda}{\beta} - \frac{1-\theta}{\alpha}\right)v_K(p)\ .
\end{aligned} \tag{2}$$

In other words, there are nonnegative a, b, and some scalar c, such that,

$$v_G(p) - v_H(p) \geq 0 \iff a v_G(p) - b v_H(p) \geq c v_K(p)\ , \tag{3}$$

where c can w.l.o.g be assumed to be nonnegative as well, otherwise the analogue equivalence, using $v_H(p) - v_G(p)$, can be employed. First it is argued that neither a nor b can be zero, on account of Richness. If $b = 0$ let p be a lottery such that $(p, H) \succ (p, G) \succ p \succ (p, K)$, then $v_G(p) - v_H(p) < 0$ but $a v_G(p) - b v_H(p) = a v_G(p) \geq 0 \geq c v_K(p)$. Otherwise if $a = 0$ let p' be a lottery such that $(p', K) \succ (p', G) \succ (p', H) \succ p'$, hence $v_G(p') - v_H(p') \geq 0$ but $a v_G(p) - b v_H(p) = -b v_H(p) < 0 \leq c v_K(p)$. We conclude that a and b are strictly positive.

It is next proved by negation that c must be zero. Suppose on the contrary that $c > 0$, and assume first that $a \leq b$. Let p be a lottery such that $(p, K) \succ (p, H) \succ$

$(p, G) \succ p$ and p' a lottery such that $(p', K) \succ (p', G) \succ (p', H) \succ p'$. Then there exists a mixture of p and p', denote it p_0, such that $v_G(p_0) - v_H(p_0) = 0$, and still $v_K(p_0) > 0$. Since $a \leq b$ and $v_G(p_0) > 0$, then $av_G(p_0) - bv_H(p_0) \leq 0$, implying that $cv_K(p_0) > av_G(p_0) - bv_H(p_0)$ while $v_G(p_0) - v_H(p_0) \geq 0$. A contradiction to Consistent Influence (C6) is inflicted. Hence $c > 0$ and $a \leq b$ is ruled out. Suppose now that $c > 0$ and $a > b$ (recall that a and b are strictly positive). Employing Richness once more, there is a lottery q such that $(q, H) \succ (q, G) \succ q \succ (q, K)$, hence $cv_K(q) < 0$ and $v_G(q) - v_H(q) < 0$. If also $v_G(q) - \frac{b}{a} v_H(q) \geq 0$ then q induces a contradiction to Consistent Influence (C6). Otherwise, Richness implies that there is a lottery q' such that $(q', G) \succ (q', H) \succ q' \succ (q', K)$, hence there exists a proper mixture of q and q', denote it $\hat{q}$ that satisfies both $v_G(\hat{q}) - v_H(\hat{q}) < 0$ and $v_G(\hat{q}) - \frac{b}{a} v_H(\hat{q}) \geq 0$. This mixture still maintains $cv_K(\hat{q}) < 0$, and a contradiction to Consistent Influence (C6) ensues. It is thus concluded that c must be zero.

Substituting $c = 0$ in (3) yields that there are positive coefficients a and b such that for every lottery p,

$$v_G(p) - v_H(p) \geq 0 \iff av_G(p) - bv_H(p) \geq 0 \ ,$$

which by using the same arguments as above implies $a = b$. Recalling the definitions of a, b and c, and the fact that a and b are strictly positive, it is concluded that in the equivalence in (2), $\alpha = \beta$ and $\lambda = \theta \neq 0$, therefore for any nonempty group K there are coefficients $\lambda_K \geq 0$ and $\delta_K > 0$ such that for any nonempty group T which is disjoint from K, $v_{K \cup T} = \lambda_K v_K + \delta_K v_T$.

Consider the groups $\{i\}$ and $\{j\}$ and denote $v_i = v_{\{i\}}, v_j = v_{\{j\}}$. For the group $\{i, j\}$, $v_{\{i,j\}}$ is a vNM utility function, satisfying both $v_{\{i,j\}} = \lambda_i v_i + \delta_i v_j$ and $v_{\{i,j\}} = \lambda_j v_j + \delta_j v_i$. As all v-functions assign a utility of zero to x_*, these two representations must be a positive multiplication of one another, and once the same scale is chosen they are identical. Richness implies that v_i and v_j are linearly independent, hence $\lambda_i = \delta_j$ and

$\delta_i = \lambda_j$. However when the vNM utility for the group $\{j,k\}$, for instance, is considered, and represented as a combination of v_j and v_k, the coefficient of v_k is also either δ_j or λ_k, which by the same arguments are identical. Hence $\lambda_k = \lambda_i$. And this argument may be applied to any pair of individuals, to yield that for every such pair, the coefficient of each v_i in the combination representing the v-utility function for this pair is the same. In other words, there exists $\lambda > 0$ such that for any two individuals i and j, $v_{\{i,j\}} = \lambda v_i + \lambda v_j$. As $v_{\{i,j\}}$ is a vNM utility function, we may choose $\lambda = 1$ to obtain $v_{\{i,j\}} = v_i + v_j$. Together with a recursive application of the conclusion from the previous paragraph, we obtain $v_G = \sum_{i \in G} v_i$ for every nonempty G. Uniqueness up to a joint scale and a shift of u_0 is implied by the uniqueness result of Proposition 1.

4.2.2 Necessity: the axioms hold

Suppose that $\succsim$ is represented as in (ii) of Theorem 1. Assumption **C1-C4** immediately follow. For Compromise (**C5**) suppose that for a lottery p and two disjoint groups G and H, it holds that,

$$\begin{aligned} u_0(q) &\geq u_0(p) \\ u_0(p) + \sum_{j \in G} v_j(p) &\geq u_0(q) + \sum_{j \in G} v_j(q) \\ u_0(p) + \sum_{j \in H} v_j(p) &\geq u_0(q) + \sum_{j \in H} v_j(q) \ . \end{aligned}$$

It follows that,

$$\begin{aligned} \sum_{j \in G} (v_j(p) - v_j(q)) &\geq u_0(q) - u_0(p) \geq 0 \ , \quad \text{and} \\ \sum_{j \in H} (v_j(p) - v_j(q)) &\geq u_0(q) - u_0(p) \geq 0 \ . \end{aligned}$$

Hence,

$$\sum_{j \in G \cup H} (v_j(p) - v_j(q)) \geq 2(u_0(q) - u_0(p)) \geq u_0(q) - u_0(p) \ ,$$

yielding that $(p, G \cup H) \succsim (q, G \cup H)$, as required by Compromise.

For richness, let G, H and K be three distinct and pairwise-disjoint sets. Consider the zero-sum game with $\mathbb{1}_x, x \in X$ as the strategies of the first player (the maximizer), $u_G - u_H, u_H - u_K$, and $u_K - u_0$ the strategies of the second player, and payoffs which are the multiplication of the strategies played. The first condition for the diversification of the utilities implies that for every possible mixed strategy of the second player there is strategy of the first player that ensures him a strictly positive payoff. Hence, by the Minimax Theorem, there is a lottery $p \in Y$ such that $p \cdot (u_G - u_H) > 0$, $p \cdot (u_H - u_K) > 0$, and $p \cdot (u_K - u_0) > 0$, delivering the first set of preferences stated in Richness. A symmetric argument, employing the second condition in the definition of diversified utilities, yields the second set of Richness preferences.

Lastly, for Consistent Influence (**C6**), let G,H and K be pairwise-disjoint, and suppose that $(p, G) \succsim (p, H)$. According to the representation assumed, it follows that $\sum_{j \in G} v_j(p) \geq \sum_{j \in H} v_j(p)$, yielding that $\sum_{j \in G \cup K} v_j(p) \geq \sum_{j \in H \cup K} v_j(p)$, as the sets are pairwise-disjoint. Adding $u_0(p)$ to both sides of this inequality delivers the required preference.

4.3 Proof of Proposition 2

It is shown that sub-additivity as in the proposition holds for every pair of lotteries p and q, for every marginal contribution to a group $S \subseteq I \setminus \{i, j\}$. Fix such p, q, and S. Note first that if both $v_i(q) > v_i(p)$ and $v_j(q) > v_j(p)$, then neither i, j nor $\bar{ij}$ can contribute to any group, as only positive contributions are accounted for, and the inequality trivially holds. Now examine the case in which both $v_i(p) > v_i(q)$ and $v_j(p) > v_j(q)$. If $\bar{ij}$ contributes zero to S (namely, $\bar{ij}$ does not swing S) then the inequality for p, q and S is trivially true. Otherwise, if $\bar{ij}$ swings S, then the corresponding marginal contribution to $\bar{B}_{\bar{ij}}(p,q)$ is $\frac{1}{2^{|N|-2}}$. It is shown that the sum of marginal contributions, of i to S and to $S \cup \{j\}$, and of j to S and to $S \cup \{i\}$, is also $\frac{1}{2^{|N|-2}}$. If both $w_{p,q}(S \cup \{j\}) = 0$ and $w_{p,q}(S \cup \{i\}) = 0$, then i swings $S \cup \{j\}$ and j swings $S \cup \{i\}$, and both do not swing S. Therefore each marginally contributes $\frac{1}{2^{|N|-1}}$, adding up to $\frac{1}{2^{|N|-2}}$. Otherwise, if both $w_{p,q}(S \cup \{j\}) = 1$ and $w_{p,q}(S \cup \{i\}) = 1$ then each of i and j swings S, and none swings S with the other, so that their marginal contributions are again $\frac{1}{2^{|N|-1}}$ each and sum to $\frac{1}{2^{|N|-2}}$. If $w_{p,q}(S \cup \{j\}) = 1$ and $w_{p,q}(S \cup \{i\}) = 0$ then i does not swing S nor $S \cup \{j\}$, but j swings both S and $S \cup \{i\}$, twice delivering a contribution of $\frac{1}{2^{|N|-1}}$ hence altogether $\frac{1}{2^{|N|-2}}$.

If $v_i(p) > v_i(q)$ and $v_j(q) > v_j(p)$, then only i can be a swinger. If $\bar{ij}$ contributes zero (namely, $\bar{ij}$ does not swing S) then the inequality for p, q and S holds trivially. If $\bar{ij}$ swings S, yielding a marginal contribution of $\frac{1}{2^{|N|-2}}$, then it must be that i swings both S and $S \cup \{j\}$ (as j only adds to the desirability of q over p, having $v_j(p) - v_j(q) < 0$). Hence i contributes a total of $\frac{1}{2^{|N|-2}}$. The symmetric case, switching i and j, is analogue.

The above is true for every $S \subseteq I \setminus \{i,j\}$, therefore for every pair of lotteries p and q, $\bar{B}_{\bar{ij}}(p,q) \leq B_i(p,q) + B_j(p,q)$, and the proof for the average follows.

4.4 Proof of Proposition 3

Denote by u_i the vNM utility that represents $\succsim^i$ over Y. Suppose that (ii) of Theorem 1 holds, with v_i the vNM utility subjectively ascribed to individual i by Individual Zero.

Assume first that v_i represents $\succsim^i$. If both $x \succ y$ and $x \succ^i y$, then equivalently, $u_0(x) > u_0(y)$ and $v_i(x) > v_i(y)$, which immediately implies that $u_0(x) + v_i(x) > u_0(y) + v_i(y)$. Hence $(x, \{i\}) \succ (y, \{i\})$.

Now suppose that whenever $x \succ y$ and $x \succ^i y$, it also holds that $(x, \{i\}) \succ (y, \{i\})$. By this and (b) it follows that $u_{\{i\}} = \lambda_0 u_0 + \lambda_i u_i + \tau_i$, for $\lambda_0, \lambda_i \geq 0, \lambda_0 + \lambda_i > 0$ (see De Meyer and Mongin [5]). Normalizing $u_0(x_*) = u_{\{i\}}(x_*) = 0$ yields $\tau_i = -\lambda_i u_i(x_*)$, so that for x^* it holds that $u_{\{i\}}(x^*) = \lambda_0 u_0(x^*) + \lambda_i u_i(x^*) - \lambda_i u_i(x_*)$. Applying (a) and the fact that $u_{\{i\}}(x^*) = u_0(x^*)$ by **C0**, it follows that $\lambda_0 = 1$, so that $u_{\{i\}} = u_0 + \lambda_i u_i + \tau_i$.

Richness (**C7**) implies, in particular, that there are p and q such that $u_{\{i\}}(p) > u_0(p)$ and $u_0(q) > u_{\{i\}}(q)$, hence $\lambda_i \neq 0$. Finally, (ii) of Theorem 1 means that $u_{\{i\}} = u_0 + v_i$, therefore $u_0 + v_i = u_0 + \lambda_i u_i + \tau_i$, yielding $v_i = \lambda_i u_i + \tau_i$. Namely, v_i represents $\succsim^i$.

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