Economic-consistency and belief-consistency

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A well known puzzle

• n individuals, seated around a table, are wearing hats

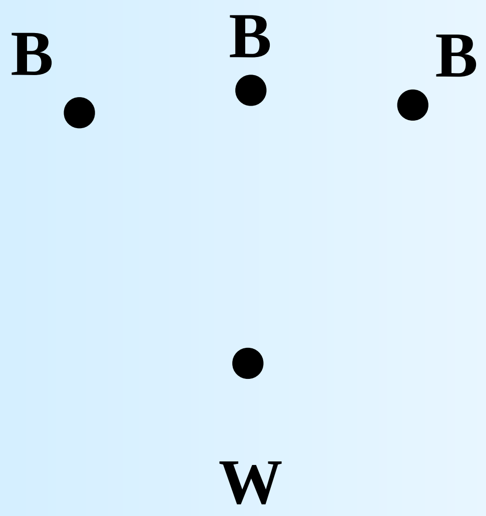
• Each can see all others' hats but not his own

• An outsider announces: "Your hats are either white or black; at least one is white.

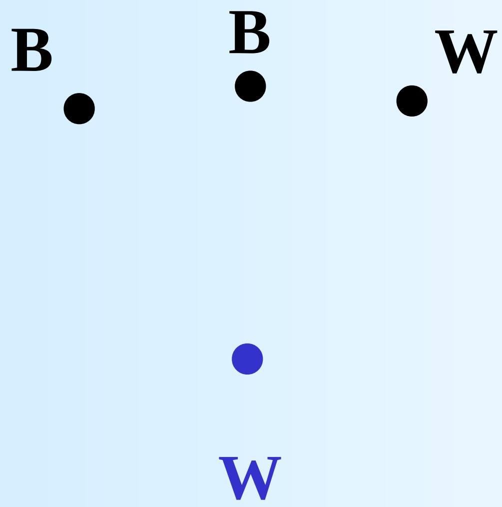
• I will start to count. After each number you will have the opportunity to raise your hand. You may do so only if you know the color of your hat."

• When, for the first time, will any individual raise her hand?

The case of 4 individuals – 1W3B



The case of 4 individuals – 2W2B



BBBW

BBWW

BBWB

BWWW

BWWB

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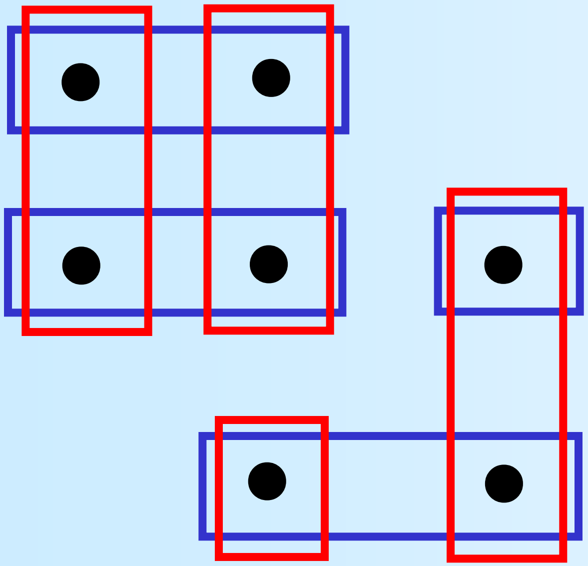
Two agents



Π_1 -

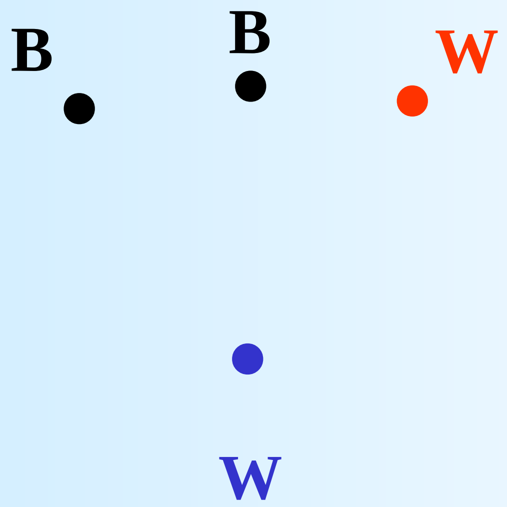
Π_2 -

Common prior and posteriors

Robert J. Aumann, (1998), Common Priors: A Reply to Gul
Econometrica, Vol. 66.

We agree with Gul that the above development of common priors is essentially dynamic, that it depends on the idea of acquiring information and using it to update one's probabilities. And though we consider this perfectly legitimate and intuitive, we also agree that it *would* be desirable to characterize common priors and/or the CPA directly in terms of the "current," posterior probabilities, without any reference, either implicit or explicit, to any prior stage.[13]

The case of 4 individuals – 2W2B



BBBW

BBWW

BBWB

BWWW

BWWB

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Posteriors and priors

	ω_1	ω_2	ω_3	ω_4	ω_5
posteriors	1/3	2/3	1/5	2/5	2/5

A prior (a1/3, a2/3, b1/5, b2/5, b2/5)

=

a(1/3, 2/3, 0, 0, 0) + b(0, 0, 1/5, 2/5, 2/5)

$a, b \geq 0$

$a + b = 1$

A *prior* of an agent is a convex combination of her posteriors.

A type space for a set of agents I

$(\Omega, (\Pi_i)_{i \in I}, (t_i)_{i \in I})$

Π_i – partition of i.

$t_i : \Omega \rightarrow \Delta(\Omega)$, $t_i(\omega)$ is a distribution over Ω

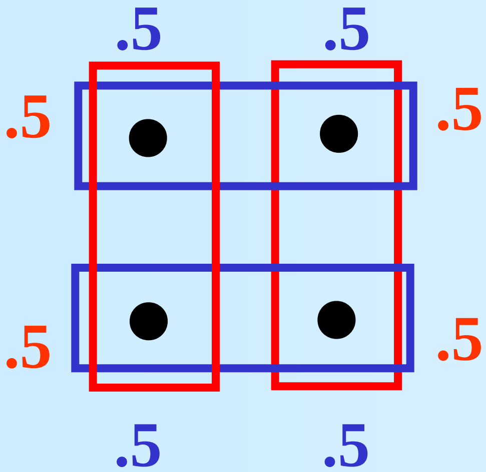
$t_i(\omega)(\Pi_i(\omega)) = 1$

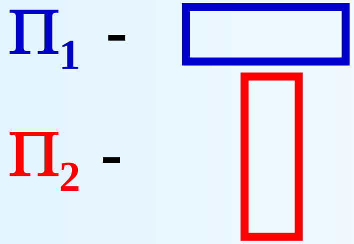
$\omega' \in \Pi_i(\omega) \Rightarrow t_i(\omega') = t_i(\omega)$

$P_i = \mathrm{Conv}\{t_i(\omega) \mid \omega \in \Omega\}$ = i's priors

$\cap_{i \in I} P_i$ – the set of *common priors*

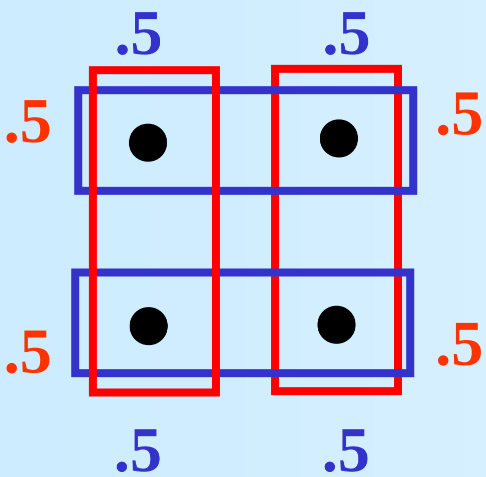
Example



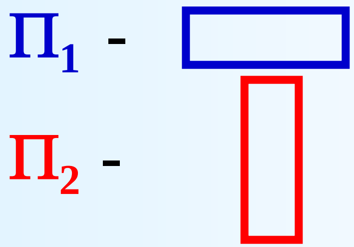


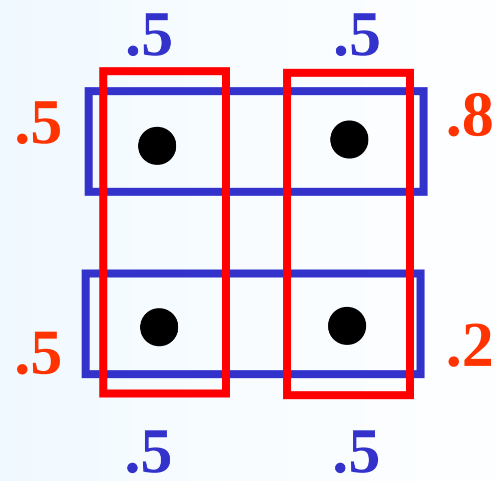
- prior of 1:
- Prior of 2:
- There is common prior

Example cont.



There is common prior





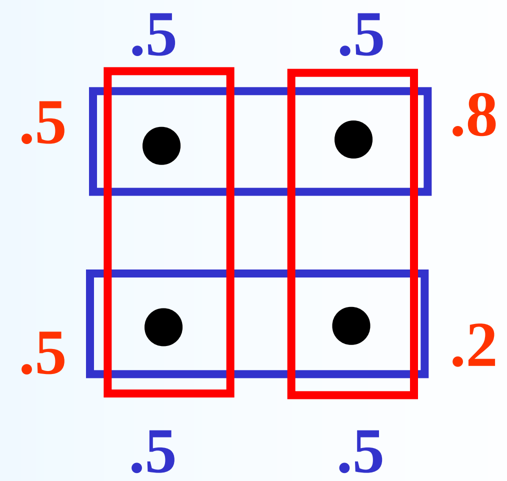
There is **no** common prior

Options – state-contingent trade

Π_1 -

Π_2 -

There is **no** common prior



\$3 ●	-\$2 ●
-\$4 ●	\$5 ●

State-contingent trade (bet)

Agent I (maximizer) is paid by Agent II (minimizer)

The case of n agents

There exists a common prior
if and only if
there are no $(f_i : \Omega \rightarrow R)_{i \in I}$
such that $\sum_i f_i = 0$
and $E_i f_i > 0$ for each $i \in I$

This is an <u>agreeable bet</u> – after they are privately informed (interim)

Its existence points to a behavioral-inconsistency (or economic-inconsistency)

- **So far we dealt only with finite spaces.**

- **Why infinite spaces?**

- **Is there any significant difference between finite and infinite cases?**

The e-mail game – a motivation

· One of two games is randomly chosen. Player I knows and player II doesn’t.

. If the game is A then player 1's computer automatically sends a message to player II's computer; if the game is B, then no message is sent.

• If a computer receives a message then it automatically sends a confirmation.

• This is so not only for the original message but also for the confirmation, the confirmation of the confirmation, and so on.

The e-mail game, cont.

- **The protocol is designed to send confirmations because there is a positive probability that any given message does not arrive at its intended destination.**

- **If a message does not arrive then the communication stops.**

- **At the end of the communication phase each player's screen displays the number of messages that his machine has sent.**

The information structure

There are infinitely many states:

$\{(q,q), (q+1,q), \mid q=0,1,2,\ldots\}$

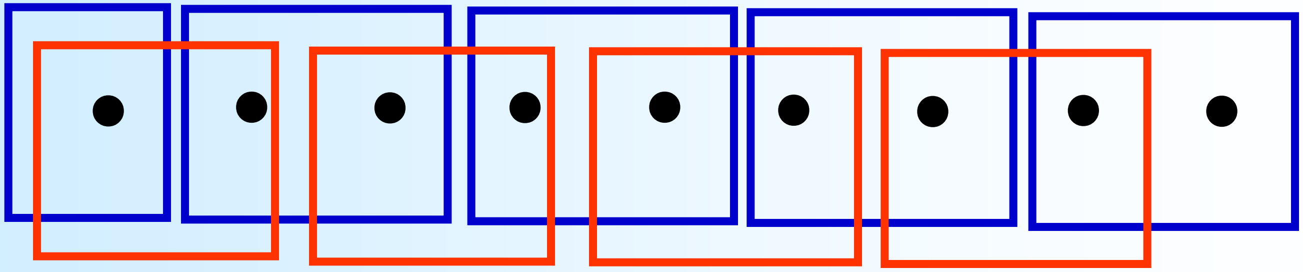
number of messages sent by player I

number of messages sent by player II

The information structure

There are infinitely many states:

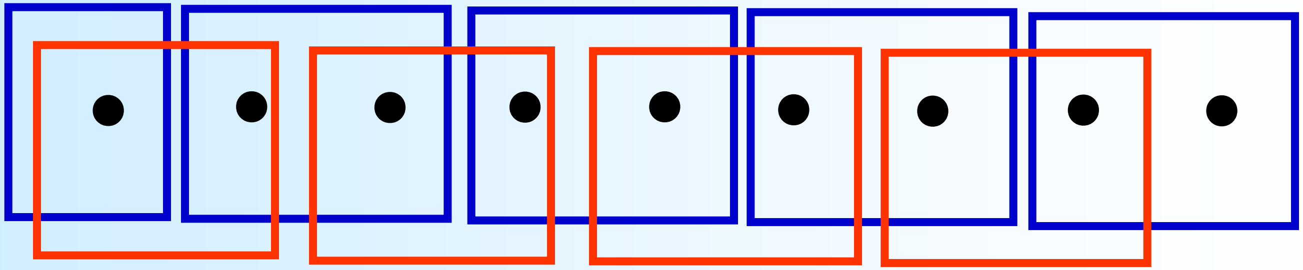
$\{(q,q), (q,q+1), \mid q=0,1,2,\ldots\}$



Private beliefs and common prior

Suppose the posteriors are all uniform within the partition elements. Is there a common prior (CP)?

NO

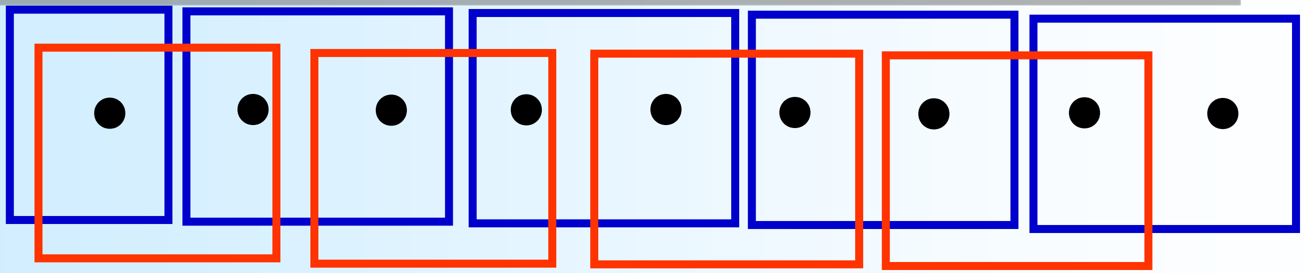


Beliefs and common prior

Suppose the posteriors are all uniform within the partition elements. Is there a common prior (CP)? NO

Can you find an agreeable trade?

YES



Infinite model

Theorem: In finite or infinite models, if CP exists then there is no agreeable bet.

~~**Theorem: If there is no agreeable bet, then CP exists**~~

Agreeable trade

An interim <u>agreeable trade</u> is bounded

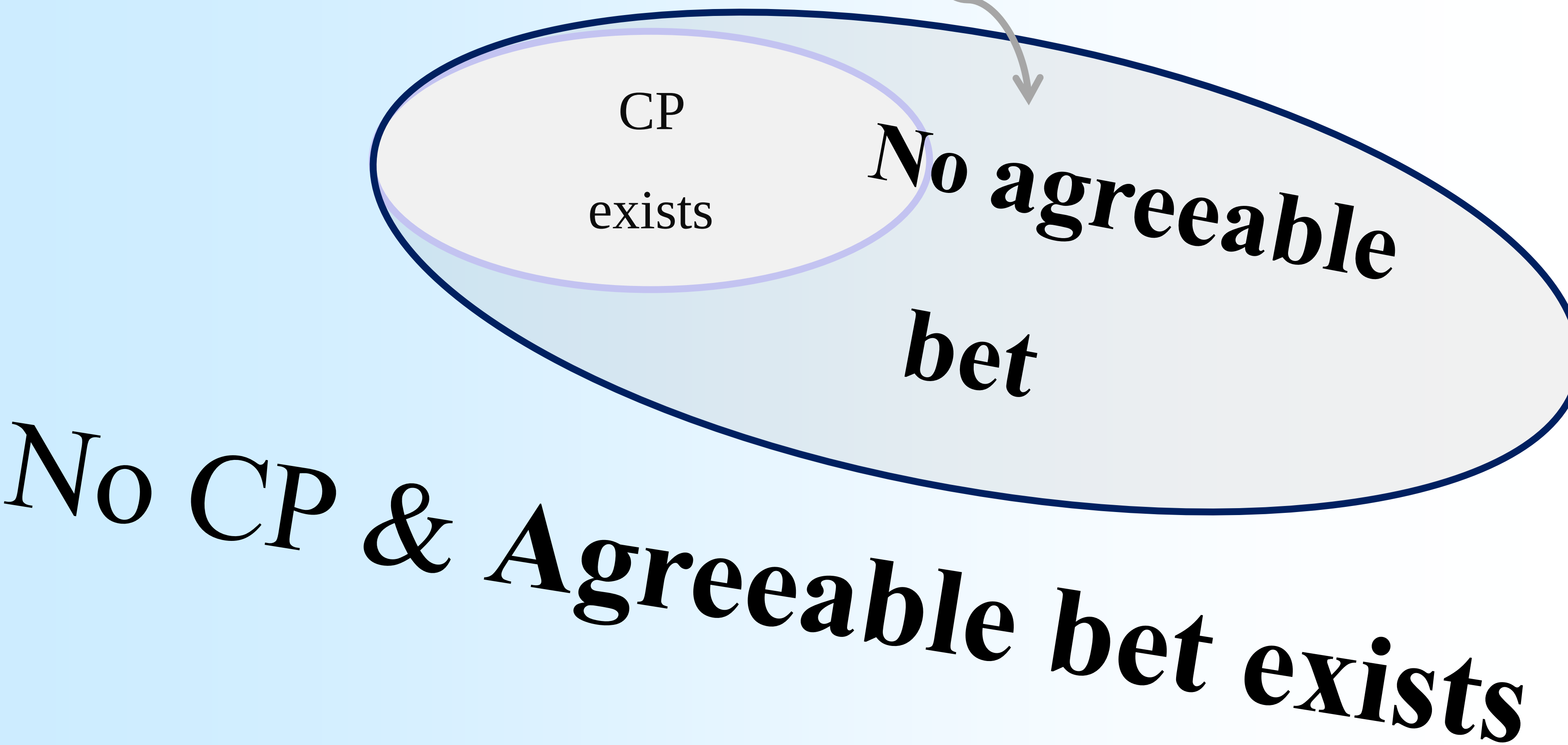
$$(f_i : \Omega \to \mathrm{R})_{i \in I}$$

$$\text{such that } \sum_i f_i = 0$$

$$\text{and } E_i f_i > 0 \text{ for each } i \in I$$

The objective of this work

Identify when No CP and No agreeable trade can coexist



The problem

In what cases no CP implies the existence of agreeable bet?

ε-Priors

Definition: an ε- prior of player *i* is …

Definition: an ε- common prior is …

Fix $\pi_i \in \prod_i$. Find the maximum of P$^{\varepsilon}(\pi_i)$.

Denote it by $d(\varepsilon, \pi_i)$.

Now check $\frac{d(\varepsilon, \pi_i)}{\varepsilon}$ as ε shrinks to 0.

Main result

Beliefs-consistency: $\lim \frac{d(\varepsilon,\pi_i)}{\varepsilon}$ **is unbounded for at least one cell** $\pi_i \in \Pi_i$ **.**

<u>Theorem:</u> In a countable space, $\lim \frac{d(\varepsilon,\pi_i)}{\varepsilon}$

is bounded for every information set $\pi_i \in \Pi_i$

if and only if an agreeable bet exists.