A new book on extrapolation methods

Practical Extrapolation Methods
Theory and Applications

by Avram Sidi

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This state-of-the-art monograph presents an in-depth *unified, comprehensive,* and *up-to-date* account of the most effective *nonlinear* scalar extrapolation methods—approximately twenty in number—that exist in the literature and are in current use. It includes all important aspects of extrapolation methods, namely, derivation of methods, design of efficient algorithms for their implementation, and rigorous convergence and stability analyses. It contains many illustrative examples that make the material easy to follow and digest. It also presents numerous applications of interest.

A fundamental computational problem one faces in different scientific and engineering disciplines is that of finding limits of sequences $\{A_m\}$, which may arise in different forms. In most cases of practical interest, these sequences converge extremely slowly; therefore, one needs to compute a large number of the terms A_m to approximate $\lim_{m\to\infty} A_m$ with reasonable accuracy, and this makes their direct use very expensive computationally. In some cases, $\{A_m\}$ may even diverge, which makes their direct use irrelevant. In case of divergence, one speaks about the *antilimit* of $\{A_m\}$ instead of its limit, and the antilimit is a quantity of relevance (it may have some physical meaning, for example); it may be related to *analytic continuation,* or *Abel summability,* or *Borel summability,* etc. Generally, whether $\{A_m\}$ converges or not, suitable *extrapolation methods* (equivalently, *convergence acceleration methods* or *sequence transformations*) can be applied to obtain the limit or antilimit of $\{A_m\}$ with high accuracy by using the terms $A_1, A_2, \ldots, A_s$, where s is small.

This book has the following unique features that distinguish it from earlier literature:

- It covers many different families of sequences, most of which not treated in previous literature, that arise in practical applications. In this sense, it has a larger scope.
- It treats divergent sequences *on an equal footing* with convergent ones for the first time, providing illuminating discussions of their antilimits simultaneously.
- It provides complete proofs of many of the theoretical results; the reader is referred to the literature only in a few cases.
- It places great emphasis on the asymptotic analysis of A_m as $m \to \infty$. Theorems and simple recipes are given for the first time by which one can deduce the form of the asymptotic expansion of A_m; this helps the user to decide easily which extrapolation method to employ.
- The issue of stability is formalized and treated in detail for the first time. The conclusions drawn from the analysis of stability are used to devise strategies—denoted *arithmetic progression sampling* (APS) and *geometric progression sampling* (GPS) in the book—that enable the relevant methods to achieve high accuracy in finite-precision arithmetic in a reliable manner.

This book is meant to serve (i) as a reference for mathematicians interested in the theory and practice of extrapolation methods, (ii) as a practical guide/manual for applied scientists and engineers who wish to employ speed-up methods in the solution of difficult computational problems, and (iii) as a textbook for advanced undergraduate and graduate level courses in convergence acceleration. Most of the required mathematics used in this work and not covered in standard curricula is summarized in several appendices that contain other useful material as well.

For the complete table of contents and part of Chapter 1, see the PDF file at
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