Workshop on Approximation Theory - Tel-Aviv, Feb. 20-21, 2002

Abstracts

Speaker: Bruce L. Chalmers, UC Riverside

Title: An n-Dimensional Hahn–Banach Extension Theorem and Minimal Projections

Abstract: Let $\tilde{T} = \sum_{i=1}^{n} \tilde{u}_i \otimes v_i : V \to V = [v_1, ..., v_n] \subset X$, where $\tilde{u}_i \in V^*$ and X is a Banach space. Let $T = \sum_{i=1}^{n} u_i \otimes v_i : X \to V$ be an extension of $\tilde{T}$ to all of X (i.e., $u_i \in X^*$) such that T has minimal (operator) norm. (E.g., if $\tilde{T} = I$, T is a minimal projection from X onto V.) Then $u := u_1, ..., u_n$ is given by the formula ($v := v_1, ..., v_n$)

$$ext_v(u) = Mv$$

for some $n \times n$ matrix M and where the notion of a v-extremal ("ext_v") of u is properly defined (i.e., $ext_v(u) := \int \alpha\, ext(\alpha \cdot u)\, d\mu$, where μ is a nonnegative measure on $\{\alpha \in S(V^*) : \|\alpha \cdot u\| = \|T\|\}$. ($L(\alpha) := \|\alpha \cdot u\|$ ($\leq \|T\|$) is the analogue of the classical Lebesgue function of T.) The formula above leads in many important cases to a simple geometric interpretation of minimal projections. Furthermore, by applying this formula to the case $X = L^p$, we obtain a *linear* n-dimensional analog of the Hölder equality condition

$$\frac{1}{p}\, u' \cdot Mv = \frac{1}{q}\, u \cdot Mv',$$

wherever v is differentiable.

As applications we determine minimal projections from $L^p[-1,1]$ onto π_{n-1} (the $(n-1)$-st degree algebraic polynomials), $1 \leq p \leq \infty$, the absolute projection constant of ℓ_n^p, and the n-dimensional symmetric space with maximal absolute projection constant. Several aspects of this work are joint in particular with F. T. Metcalf, G. Lewicki, and B. Shekhtman.

Finally, we indicate how the theory extends to minimal shape-preserving projections and has resulted in work joint with M. P. Prophet and D. Leviatan.

Speaker: Wolfgang Dahmen, Aachen

Title: Sparse Approximation of Nonlinear Functions of Multiscale Expansions

Abstract: This talk is concerned with the estimation and evaluation of wavelet coefficients of *nonlinear* functions of multiscale expansions. The main result is to show that certain sequence spaces that can be used to measure the sparsity of tree type arrays of wavelet coefficients are stable under certain nonlinear mappings identified by some basic requirements. We indicate how these results can be used to facilitate the sparse evaluation of wavelet coefficients of compositions at asymptotically optimal computational cost and how this enters the adaptive solution of nonlinear variational problems. Furthermore, the basic requirements are verified for several concrete choices of nonlinear mappings.

Speaker: Shai Dekel, Tel-Aviv

Title: On the Relations between Piecewise Polynomial and Rational Approximation in $L_p(R^2)$

Abstract: In the univariate case there are certain equivalences between the nonlinear approximation methods that use piecewise polynomials and those that use rational functions. It is known that for certain parameters the respective approximation spaces are identical and may be described as Besov spaces. The characterization of the approximation spaces of the multivariate nonlinear approximation by piecewise polynomials and by rational functions is not known. In this work we compare between the two methods in the bivariate case. We show some relations between the approximation spaces of piecewise polynomials defined on n triangles and those of bivariate rational functions of total degree n which are described by n parameters. Thus we compare two classes of approximants with the same number cn of parameters. We consider this the proper comparison between the two methods.

Speaker: Ron DeVore, Univ. South Carolina

Title: Compression and Encoding of Digital Terrain Data

Abstract: Digital Terrain maps are now available for the entire earth at varying resolutions. They are used for navigation, emergency preparedness and other applications. There is a need to efficiently compress these maps, i.e. to represent them as a bitstream with few bits. The mathematical problem is how to encode surfaces. The proper metric seems to be the Hausdorff metric or some variant. We shall discuss preliminary theorems and algorithms for approximation of surfaces in the Hausdorff metric. This is joint work originally inspired by conversations with many mathematicians including Nira Dyn, Peter Schroeder, and Vladimir Temlyakov. The theorems proved here were obtained with Albert Cohen, Wolfgang Dahmen, and Ingrid Daubechies.

Speaker: Zeev Ditzian, Univ. Alberta, Edmonton

Title: Inequalities of Kolmogorov and Wirtinger Type

Abstract: We show that a Kolmogorov-type inequality follows from a Jackson-type and a non-matching Bernstein-type inequality. We show that a Wirtinger-type inequality for a given self-adjoint operator follows from the boundedness of the Cesaro summability of the eigenfunctions of that operator. Examples will be given.

Speaker: Manfred von Golitschek, Würzburg

Title: Penalized Least Squares Approximation

Abstract:K functionals play an important role in approximation theory. In their simplest form, let X_0 and X_1 be Banach spaces of real-valued functions, with X_1 continuously embedded in X_0. Then the K functional for $f \in X_0$ is defined by

$$K(f,\lambda) := \inf_{g\in X_1} \left\{ \|f-g\|_{X_0} + \lambda \|g\|_{X_1} \right\}, \quad \lambda > 0.$$

Closely related to K functionals, but easier to study theoretically and numerically, are the **quadratic functionals**

$$Q(f,\lambda) := \inf_{g\in X_1} \left\{ \|f-g\|^2_{X_0} + \lambda \|g\|^2_{X_1} \right\}, \quad l > 0, \tag{1}$$

where X_0 and $X_1 \subset X_0$ are also linear spaces of real-valued functions, while $\|\cdot\|_{X_0}$ and $\|\cdot\|_{X_1}$ are semi-norms induced by semi-definite inner products $\langle\cdot,\cdot\rangle$ on X_0 and $[\cdot,\cdot]$ on X_1. If for $\lambda > 0$ and each $f \in X_0$ there exists a unique minimal $g = g_{f,\lambda} \in X_1$ in (1), then $Q(\cdot,\lambda)$ defines a *linear operator* $\mathbf{Q}_\lambda : X_0 \to X_1$ by $\mathbf{Q}_\lambda f := g_{f,\lambda}$.

In my talk I will discuss the approximation properties of the linear operator $\mathbf{Q}_\lambda$ for various linear spaces $X_0 \subseteq L_2(\Omega)$ where Ω is a real interval or a compact subset of R^2. The inner product on X_0 will be either $\langle f,g\rangle = \int_\Omega fg$ or $\langle f,g\rangle = \sum_{i=1}^m f(t_i)g(t_i)$ with m fixed points in Ω. The linear space $X_1 \subset L_2(\Omega)$ will be either the Sobolev spaces $W_2^2(\Omega)$ or some finite dimensional linear subspace of $W_2^2(\Omega)$, while $[f,g] = \int_\Omega f''g''$ if $\Omega = [a,b]$ is a real interval, or $[f,g] = \int_\Omega \Delta f \Delta g$ with the Laplace operator Δ if Ω is the closure of a bounded open set in R^2 with piecewise differentiable boundary $\partial\Omega$ of finite length.

Speaker: Panagiotis Kaklis, Athens

Title: Spatial Geometric Hermite Interpolation

Abstract: This work deals with the problem of constructing spatial Hermite interpolants subject to geometric boundary conditions, i.e., given Frenet frame, curvature and torsion.

A two-parameter family of quintic solutions is constructed and its properties are classified according as a boundary curvature is zero and a boundary binormal is orthogonal to the unit tangent of the other boundary Frenet frame.

A technique is proposed for selecting the free-parameter pair by minimizing the L_2-norm of the 2nd/3rd parametric derivative of the Hermite interpolant. Its performance is tested again data taken from a helix, a bicyndrical and an exponential curve.

Finally, some preliminary results will be presented, dealing with our current work on exploiting these Hermite interpolants for solving the global interpolation problem: construct a third-order Frenet continuous curve, interpolating a given pointset and satisfying geometric boundary conditions.

Speaker: Viktor N. Konovalov, Institute of Mathematcs, Kyiv

Title: Influence of the Shape of Functions on the Degree of Approximation

Abstract: Let B_p be unit ball in the space L_p, $1 \le p \le \infty$, of functions $x : I \to R$ defined on a finite interval I. Let $\Delta_+^s B_p$, $s \in N_0$, be the subclass of functions $x \in B_p$ such that the kth divided differences of x are nonnegative on I for each $k = 0, \dots, s$. It is evident that the functions $x \in \Delta_+^0 B_p$ are nonnegative, the functions $x \in \Delta_+^1 B_p$ are nonnegative and nondecreasing, the functions $x \in \Delta_+^2 B_p$ are nonnegative, nondecreasing, and convex, etc. For all $n, s \in N_0$, $1 \le q \le p \le \infty$, we obtain two-sided estimates for the Kolmogorov n-widths $d_n(\Delta_+^s B_p)_{L_q}^{kol}$, linear n-widths $d_n(\Delta_+^s B_p)_{L_q}^{lin}$, Alexandrov n-widths $d_n(\Delta_+^s B_p)_{L_q}^{alx}$ and pseudo-dimensional n-widths $d_n(\Delta_+^s B_p)_{L_q}^{psd}$ of the classes $\Delta_+^s B_p$ in the space L_q. It will be shown that the degree of approximation essentially depends on the parameter s, i.e., on the shape of the functions.

Speaker: Paul Nevai, The Ohio State University

Title: Characterization of Orthogonal Polynomials (from George Pólya to Barry Simon)

Abstract: It all started with George [then György] Pólya who asked a 17 year old student Gabor [then Gábor] Szegő to study determinants of Toeplitz forms. This happened in 1912 or 1913. The rest is history. Szegő not only ended up discovering [inventing?] orthogonal polynomials on the unit circle and laying the foundations of H^p spaces, but he and other distinguished mathematicians such as Akhiezer, Freud, Geronimus, Kolmogorov, M. G. Krein, V. I. Smirnov, Shohat, have finished the "job" within 40 years or so.

The job consisted of complete characterizations of orthogonal polynomials, the corresponding measures, and the related linear recurrences, in terms of each other. Then the subject was dormant for about 20 years or so although some interesting results and observations were published here and there such as Baxter's theorem in 1961. Then, starting with 1976, a busy decade followed when history was completely rewritten in the works of Máté, Nikishin [a bad guy but then one should let the dead rest in peace], Rakhmanov, Totik, and, with modest contributions, by yours truly. After these astonishing results, the books of history were closed again, at least as far as the "job" was concerned.

Then, out of the blue sky, S. Khrushchev came on the scene of orthogonal polynomials who is one single masterpiece rewrote history and started a new era. It is probably a safe bet to say that S. Khrushchev's work is equal to the best ever produced in the general theory of orthogonal polynomials. He was later joined by Golinskii. At the same time, the big guns of Barry Simon and Co., including Hundertmark and Killip, became interested in the subject matter as well. As we all know, once Simon comes, no earth is left unturned. In a short period of time Khrushchev–Golinskii, and Simon and Co. have produced an amazing number or deep and truly unexpected results. My talk will try telling part of the story.

My dear friend, Dany, yom huledet sameach! I hope that the next 60 years will be equally fruitful, especially when you had already 60 years to learn how to live and to figure it out what really matters in life and what is that one can safely ignore.

Speaker: Konstantin Oskolkov, Univ. South Carolina

Title: Gridge Approximation and Radon Compass

Abstract: The intention of this talk is to present results obtained in a recent joint paper by V. E. Maiorov, K. I. Oskolkov, V. N. Temlyakov.

Greedy algorithms in ridge approximation will be considered. The neologism gridge appeared as a composite of the terms greedy and ridge. Such algorithms are known in Mathematical Statistics as projection pursuit regression, and in Signal Processing as matching pursuits. They consist in step-wise, iterative constructions of linear combinations of ridge functions (plane waves). On each new step, one more ridge function is added to the preceding combination. This ridge function is selected greedily, i.e. optimally within the dictionary of ridge functions.

The resulting linear combinations are considered as approximants of a multivariate function, which is assumed to be known. The efficiency of a gridge algorithm is measured in the norm of the Hilbert space $L_2(B)$. The latter is the set of square-integrable functions in the unit Euclidean ball B^d, with regard to Lebesgue measure.

The main qualitative result of this paper is that gridge approximation is not order saturated on functional classes of finite smoothness. Quantitatively, for such classes the gridge algorithm is equally efficient with the best N-term ridge approximation, or classical algebraic polynomial approximations of the corresponding orders.

The central difficulty consists in the search of the optimal direction of propagation of ridge functions, which is a non-linear optimization problem. The latter is solved by maximizing the Radon compass, a positive continuous function on the unit sphere. It is shown that the Radon compass is uniformly continuous for the sequence of steps of the gridge algorithm. Under mild additional assumptions concerning the target function, the Radon compass is uniformly differentiable.

Speaker: Igor A. Shevchuk, University of Kyiv

Title: Coconvex Polynomial Approximation

Abstract: Let Δ^2 be the set of convex on $[-1,1]$ functions f, Π_n be the set of algebraic polynomials of degree not exceeding n, and

$$E_n^{(2)}(f) := \inf_{p_n \in \Pi_n \cap \Delta^2} \|f - p_n\|_{C[-1,1]},$$

be the error of the best uniform approximation of $f \in \Delta^2$ by convex polynomials.

In 1983 Leviatan proved that if $f \in \Delta^2$, then

$$E_n^{(2)}(f) le \omega_2^{\varphi}\left(f, \frac{1}{n}\right) le \omega_2\left(f, \frac{1}{n}\right), \quad n \geq 1,$$

where $\omega_k^{\varphi}(f,t) \quad (\omega_k(f,t))$ are Ditzian-Totik (ordinary) moduli of smoothness, and c is an absolute constant.

We will present a series of new estimates for the convex uniform polynomial approximation, as well as for the piecewise convex (so-called coconvex) approximation. They are all corollaries of the following Theorem 1. Theorem 1 was recently proved by Leviatan and the author. Here for simplicity we formulate it for convex approximation only, that is not for the coconvex case.

Theorem 1. *For every $k \in N$ there are two constants $c^* = c^*(k)$ and $c = c(k)$, such that if S is a piecewise polynomial of degree $k-1$ on the Chebyshev partition $x_j = \cos(j\pi/n), \quad j = 0, \ldots, n,$ of $[-1,1]$, and $S \in \Delta^2$, then*

$$E_{c_* n}^{(2)}(S) \leq \omega_k^{\varphi}\left(S, \frac{1}{n}\right) \leq \omega_k\left(S, \frac{1}{n}\right), \quad n \geq 1.$$

Speaker: Jozsef Szabados, Budapest

Title: Markov-Bernstein Factors for Multivariate Polynomials

Abstract: We consider the tangential derivative of bivariate polynomials on the curves $y = x^{\alpha}$ and $y = e^x$, and obtain (mostly sharp) Bernstein and Markov type estimates. The Bernstein type inequalities will show that the unusually high norm estimates hold only in "small" intervals, while in general the derivative is much smaller.

Speaker: Eitan Tadmor, UCLA

Title: Adaptive Kernels for the High Resolution Recovery of Piecewise Smooth Data from its Spectral Information

Abstract: We discuss the reconstruction of piecewise smooth data from (pseudo-) spectral information. While spectral projections enjoy superior resolution with globally smooth data, the presence of jump discontinuities is responsible for spurious O(1) Gibbs oscillations and an overall deterioration to the unacceptable first-order convergence rate of spectral projections. The purpose is to regain the superior (in fact – exponential) accuracy in the piecewise smooth case. This is achieved in two complementing steps. First, a localization step using a novel detection procedure which identifies both the location and amplitudes of finitely many edges. It is followed by a second step of mollification - we present a two-parameter family of spectral mollifiers which recover the data between those edges with exponential accuracy.

The ubiquitous one-parameter, finite-order mollifiers are based on dilation. In contrast, our family of two-parameter spectral mollifiers achieve their high resolution by an intricate process of high-order cancelation. Accurate recovery of piecewise smooth data is carried out in the direction of smoothness, and adaptivity is responsible for the high resolution. We conclude with examples for applications in Computational Fluid Dynamics (- formation of shocks), image and geophysical data processing.

Speaker: Vladimir Temlyakov, Univ. South Carolina

Title: Lower estimates in best m-term approximation.

Abstract. We will discuss how different asymptotic characteristics of function classes (compacts) can be used in estimating from below the best m-term approximation with regard to bases. We will present results for some special collections of bases: orthogonal bases, unconditional bases, greedy bases. These results can also be viewed as results on best basis selection problem. Some open problems will be mentioned.
