Homework number 1.

**Question I:** Assume that $R(x)$ is strictly convex and let $f_\tau \in R^N$ be arbitrary vectors. Assume that $y_t$ has the property that $\nabla R(y_t) = -\eta \sum_{\tau=1}^t f_\tau$. Show that,

$$\arg \min_{x \in K} \left[ \sum_{\tau=1}^t f_\tau \cdot x + \frac{1}{\eta} R(x) \right] = \arg \min_{x \in K} B^R(x\|y_t)$$

**Question II:** Compute a subgradient of $f(x) = \max_{i=1,\ldots,m} |a_i^T x + b_i|$, at any point $x$. (Sufficient to find one subgradient, no need to characterize all of them.)

**Question III:** Show that in Follow The Leader (FTL) the regret is bounded by the number of changes in the best action (assuming that the losses are in $[0,1]$).

**Question IV:** Consider Follow The Perturbed Leader (FTPL) for a quadratic optimization function, i.e., $f_t(x, w_t) = \sum_{i=1}^N \sum_{j=1}^N w_{i,j,t} x_i x_j$.

Show how to use the linear FTPL for this setting. What is the regret bound? (You can use the bound shown in class of $\Omega(\sqrt{RADT})$, where $D \geq \|d_1 - d_2\|_1$, $R \geq |d \cdot s|$, and $A \geq \|s\|_1$.) (Hint: map the problem to a higher dimension.)

The homework is due in two weeks

**Research Question**

The question here are intriguing research questions (not part of the regular homework)

**Challenge 1:** There must be a simpler way to do the approximation algorithms!

**Challenge 2:** Try to derive an extension to FTPL for non-linear functions.