

Lecture 8: Information Cascading

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8.1 Introduction

Consider a game of drawing balls from a bucket. The game manager tosses a secret coin. If the outcome is heads, the manager places 3 red balls and 2 white balls, and vice versa if the outcome is tails. Each player in his turn draws a ball from the bucket, secretly and at random, and has to announce (to all other players) his guess of the outcome of the original toss.

Now, suppose that the first few players saw red balls and declared 'heads'. The first player to see a white ball, will attribute it to chance, and, given the short sequence of red balls before her draw, she will continue to declare 'heads'. The next player has no indication that any white balls were drawn, and thus even if he too will draw a white ball he will surely declare 'heads', and so on. We see that even if that vast majority of players draw white balls, they will all believe that there is a majority of red balls, because of a small number of draws at the beginning of the game.

8.1.1 Information Cascade

We would like to generalize the discussion of behaviours such as that of the players in the game we described. Informally, we define the concept of information cascade:

Definition 8.1.1. *Information Cascade (informal)*

An information cascade occurs when it is optimal for an individual, having observed the actions of those ahead of him, to follow the behavior of the preceding individual without regard to his own information.

Information cascade can also be observed in many "real-life" situations such as stock market bubbles. A key characteristic of information cascades is that they are fragile. Many small changes can end a cascade. After a cascade has started, if an individual has more precise information than their predecessors they may still rely on their own information. If public information is introduced, the cascade could end or reverse. If there is even uncertainty as to whether the underlying state has changed, the cascade could end or reverse.

8.2 A Simple Binary Model

Similarly to the game in the introduction, consider a sequence of agents:

- Each agent decides to adopt or reject a project, based on his own signals and the decisions of previous agents;
- The project has a payoff $V \in \{0, 1\}$ (with equal probability), and a cost to adopting $C = 1/2$.
- Each agent observes a signal of $X_i \in \{H, L\}$
 - H is observed with probability $p > 1/2$ if $V = 1$, with probability $1 - p$ if $V = 0$.

Information cascade in this setting:

- The 1st agent adopts if her signal is H, else rejects
- The 2nd agent adopts if both 1st adopted and his signal is H, flips coin if 1st adopted and his signal is L
- 3rd agent: if agent 1 and 2 adopted (rejected), 3rd observes a majority of H 's (L 's), and therefore also adopts (rejects) - regardless of her signal. In this case she is the first agent in the cascade. If 1st and 2nd agents disagreed, it starts over.
- Once one agent follows regardless of his signal, all others will follow – a cascade continues indefinitely once it starts.

Two main conclusions can be drawn:

1. The most striking conclusion is an information cascade will eventually occur with probability one! For a cascade where all agents adopt the project from a certain point in time all we need is that the number of adopters to exceed the number of agents who reject by at least two in some history. A similar conclusion holds when we consider cascade where all agents reject.
2. Information cascades are fragile. If at some point an agent with a more accurate signal arrives then he may choose to ignore the actions of previous agents as they are not very informative.

8.3 Application to Finance: IPOs [6]

Consider an entrepreneur who wishes to sell his firm in an IPO. We normalize his value for the firm to be zero and let the firm's value, V , to n other investors be uniformly distributed over $[0, 1]$. Hence, it is commonly known that there is a reason for the entrepreneur to sell. This can be justified by the fact that he is under-diversified.

Suppose that investors hold binary signals which are i.i.d conditional on the asset's value. In particular:

$$\begin{aligned}\Pr [s_i = H|V = v] &= v \\ \Pr [s_i = L|V = v] &= 1 - v\end{aligned}$$

In our model, the entrepreneur is allowed to sale only one share to each investor, and the commerce of other shares does not directly affect the value of the share bought by an investor. Simple calculations reveal the following facts:

Claim 8.1.

$$\Pr [k H \text{ 's out of } n \text{ signals}] = \frac{1}{n+1}$$

Claim 8.2.

$$\Pr [at \text{ least } k H \text{ 's out of } n \text{ signals}] = 1 - \frac{k}{n+1}$$

Claim 8.3.

$$E[V|k H \text{ 's out of } n \text{ signals}] = \frac{k+1}{n+2}$$

Proof. Claim follows directly from 8.1.

For proofs of 8.1 and 8.3 see Appendix. □

8.3.1 Pricing the Issue when there is Perfect Information Sharing

Suppose that agents share the signals among them and there are exactly k 'H's. Agents would buy if and only if the price does not exceed $\frac{k+1}{n+2}$. We can solve for the optimal price by thinking about the optimal k :

$$\begin{aligned}\arg \max_k \Pr [at \text{ least } k H \text{ 's}] \cdot (\text{price assuming } k H \text{ 's}) &= \arg \max_k \left(1 - \frac{k}{n+1}\right) \left(\frac{k+1}{n+2}\right) \\ &= \arg \max_k (n+2 - (k+1))(k+1) \\ &= n/2\end{aligned}$$

This implies that the optimal price is $1/2$ and that the probability an issue will succeed is $1/2$.

8.3.2 Pricing the Issue with Information Cascades

Suppose now an agent observes only the decision of the previous investor but not his signal. How can the seller ensure that that an IPO is successful with probability one?

Notice that if some agent receives an L and buys (receives an H and sells), then no additional information is accumulated. Therefore, all future agents will also buy (sell).

Suppose that the offer price is $1/3$ and consider the first buyer. From Claim 8.3 it follows that this investor would buy even if he has a low signal. By induction, we see that all other investors would follow. Conversely, if one chooses price of $2/3$ or more then the issue will fail as the first agent is certain to refuse to buy.

Claim 8.4. *The optimal price is $1/3$.*

Proof. The proof is based on the claim that there is a significant risk of a negative cascade if the price is more than $1/3$. For details see the appendix. □

8.3.3 Remarks

The above is just an example, but several implications follow even in the more general case. For example, the under-pricing is more significant if there is more uncertainty in the prior (i.e. adding a mean preserving spread). Under-pricing increases also when the seller is risk averse.

8.4 Application to Finance: Trading in Financial Markets [1]

Recall, that in Section 8.2 we saw that in the simple binary model an information cascade occurs with probability 1. Now, if we introduce a market maker that varies the price, \hat{c} , according to the publicly available information:

$$\hat{c} = E[V|H_t]$$

$$E[V|H_t, S = L] < \hat{c} < E[V|H_t, S = H]$$

Notice that in this case players act according to their individual values and information cascading does not occur.

Somewhat more formally, consider the following model of a market, originally due to [5]:

- Noise traders buy or sell or do not trade a probability of $1/3$ regardless of the price.
- The asset takes one of two values $V \in \{0, 1\}$ with equal probabilities.
- Informed traders get private signals which are i.i.d conditional on the value of the asset.
- The history of transaction up to but not including time t (which is observed by all agents) is denoted by H_t , where h_t denotes the current transaction.
- We let $V_M^t = E[V|H_t]$ denote the expected value conditional on the available history of transactions.

- We let $V_S^t(s) = E[V|H_t, S = s]$ denote the expected value conditional on the available history of transactions and an agent's signal being s .

Claim 8.5. *In this model, an information cascade occurs with probability 0.*

First, we should introduce a formal definition of information cascade:

Definition 8.4.1. *Information Cascade (formal)*

An information cascade is a situation where the transaction at time t is determined solely by the history and not the private signal or the value of the asset:

$$\Pr[h_t|V, H_t] = \Pr[h_t|H_t] \forall h_t, V$$

Proof. An information cascade implies that the market maker does not learn from actions and hence the bid equals the ask and both equal the conditional expectation. Given the noise traders it is never the case that the market maker knows the value with certainty. Hence, a noisy signal is informative and there are agents who would buy and some agents who would sell at this price. This implies that an action conveys information which leads to contradiction. \square

While an information cascade never occurs we may observe a less striking pattern which we denote by 'herding':

8.4.1 Herding

Definition 8.4.2. *Herding*

We say that an agent engages in herd buying at time t if he buys when $V_S^0(s) < V_M^0 < V_M^t$. A herd in selling is defined in a similar way

Definition 8.4.3. *Mononocity*

A monotonic signal is a signal for which there exists a function $V(s)$ s.t. $V_S^t(s) \in [V_M^t, V(s)]$

Intuitively, when signals are monotone they could be labeled as either positive or negative. A positive signal leads to a revise upward regardless of the specific history.

Claim 8.6. *A herd behavior does not occur if the signal is monotonic.*

Proof. Suppose that a trader with a monotonic signal herd-buys at time t . Then his valuation exceeds the ask which exceeds the market expectation, i.e.:

$$V_S^t(s) > A^t > V_M^t$$

So the signal is "positive" given the history:

$$V(s) > V_M^t$$

However, since he is herd-buying, history must also be "positive":

$$V_M^0 < V_M^t < V(s)$$

Therefore it must be that the signal is “positive” even without the history:

$$\begin{aligned} V_S^0(s) &\in [V_M^0, V(s)] \\ V_S^0(s) &> V_M^0 \end{aligned}$$

Which is a contradiction to the assumption of herd-buying. □

8.4.2 A setup with herding

Given the above result it seems that for herding to occur we need to have non-monotonic signals. We consider a different version of the above market model:

$$\begin{aligned} V &\sim_{Uni} \left\{ 0, \frac{1}{2}, 1 \right\} \\ S &\in \left\{ 0, \frac{1}{2}, 1 \right\} \\ \alpha &\in \left(\frac{1}{2}, 1 \right) \\ \Pr \left[S = \frac{1}{2} | V \right] &= \begin{cases} 1 & V = \frac{1}{2} \\ 0 & \text{otherwise} \end{cases} \\ \Pr [S = 1 | V] &= \begin{cases} \alpha & V = 1 \\ 1 - \alpha & V = 0 \end{cases} \\ \Pr [S = 0 | V] &= \begin{cases} \alpha & V = 0 \\ 1 - \alpha & V = 1 \end{cases} \end{aligned}$$

We first need to verify that indeed the signal is non-monotone:

Example 1. Consider a belief where

$$\begin{aligned} \Pr \begin{bmatrix} 0 \\ \frac{1}{2} \\ 1 \end{bmatrix} &= \begin{matrix} q \\ t \\ r \end{matrix} \\ q + t + r &= 1 \end{aligned}$$

Now consider the case where t is close to one, which implies that based on these beliefs $E[V] \approx \frac{1}{2}$. Now suppose in addition that $q/r \gg 1$; this implies that $\pi = \Pr[V = 1 | V \neq \frac{1}{2}]$ is also very close to one. Consider the signal $S = 0$: Based on this signal, one knows with certainty that $V \neq \frac{1}{2}$. Hence, we have that:

$$\begin{aligned} E[V | S = 0] &= \Pr[V = 1 | S = 0] \\ &= \frac{\Pr[V = 1 \wedge S = 0]}{\Pr[S = 0]} \\ &= \frac{\pi \cdot \alpha}{\pi \cdot \alpha + (1 - \pi) \cdot (1 - \alpha)} \end{aligned}$$

While being smaller than π , this is also close to one, so $E[S|V = 0] > E[V]$.

We now prove that such a scenario is in fact likely to happen:

Claim 8.7. *There is positive probability of herding.*

Proof. Suppose that initially there many rounds with no-trades. After each no-trade the market maker thinks that this maybe due to having no information event as he was facing agents with signal $S = 1/2$. Hence, the likelihood of $v = 1/2$ increases substantially. An agent who has $S = 0$ knows that this is not the case but is not sure whether $V = 0$ or $V = 1$. So if after that we see many buy orders this agent could become more optimistic than the market maker and would buy.

□

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Appendix A

Proofs

A.1 Proofs of claims 8.1 and 8.3

A.1.1 Intuitive proofs

Claim.

$$\Pr[k H \text{'s out of } n \text{ signals}] = \frac{1}{n+1}$$

Proof. Notice that there is no importance to the order in which the signals arrive. Consider the following two experiments:

1. The entrepreneur draws a value $V \sim_{Uni} [0, 1]$, and then also each of the n agents draws $v_i \sim_{Uni} [0, 1]$. If $v_i < V$, then agent i receives a signal H , and otherwise a signal L .
2. $n+1$ points $\{x_j\}$ are drawn i.i.d. and uniformly from $[0, 1]$. The entrepreneur chooses a $j^* \sim_{Uni} [n+1]$ and then we count how many x_j 's are smaller than x_{j^*} .

It is easy to see that these experiments are essentially the same. Also from symmetry it follows that for any $k+1 \in [n+1]$

$$\begin{aligned} \Pr[k H \text{'s out of } n \text{ signals}] &= \Pr[|\{i: v_i < V\}| = k] \\ &= \Pr[|\{j: x_j < x_{j^*}\}| = k] \\ &= \Pr[\text{the entr. chooses the } k+1^{\text{th}} \text{ point}] \\ &= \frac{1}{n+1} \end{aligned}$$

□

Claim.

$$E[V | k H \text{'s out of } n \text{ signals}] = \frac{k+1}{n+2}$$

Proof. Consider what happens when we add an $n + 2^{\text{nd}}$ signal / point to each of the above two experiments:

$$\begin{aligned} \frac{k+1}{n+2} &= \Pr[\text{the new point is one of the first } k+1] \\ &= E[x_{j^*} | |\{j: x_j < x_{j^*}\}| = k] \\ &= E[V | |\{i: v_i < V\}| = k] \\ &= E[V | k \text{ H's out of } n \text{ signals}] \end{aligned}$$

□

A.1.2 Technical proofs

Claim.

$$\Pr[k \text{ H's out of } n \text{ signals}] = \frac{1}{n+1}$$

Proof.

$$\begin{aligned} \int_0^1 \Pr[k|p] \cdot \Pr[p] dp &= \int_0^1 \binom{n}{k} x^k (1-x)^{n-k} dx \\ &= \left[\binom{n}{k} \frac{x^{k+1}}{k+1} \cdot (1-x)^{n-k} \right]_0^1 + \int_0^1 \binom{n}{k} \frac{x^{k+1}}{k+1} (n-k)(1-x)^{n-k-1} dx \\ &= \int_0^1 \binom{n}{k+1} x^{k+1} (1-x)^{n-k-1} dx \\ &= \int_0^1 \Pr[k+1|p] \cdot \Pr[p] dp, \end{aligned}$$

where the transition from the second to the third expression is due to the identity

$$\binom{n}{k} \frac{(n-k)}{k+1} = \binom{n}{k+1}$$

Comparing both ends of the above sequence of equalities we realize that all the probabilities are equal, and therefore

$$\int_0^1 \Pr[k|p] \cdot \Pr[p] dp = \frac{1}{n+1}$$

□

Claim.

$$E[V | k \text{ H's out of } n \text{ signals}] = \frac{k+1}{n+2}$$

Proof.

$$\Pr[(k, n) | p] = p^k (1-p)^{n-k}$$

$Pr[(k, n)] = \int_0^1 p^k (1-p)^{n-k} dp = \frac{1}{n+1} \cdot \frac{1}{\binom{n}{k}}$ Hence:

$$\begin{aligned} E[p|(k, n)] &= \int_0^1 p \cdot \frac{Pr[(k, n)|p] \cdot Pr[p]}{Pr[(k, n)]} dp \\ &= \frac{\int_0^1 p \cdot p^k (1-p)^{n-k} dp}{\frac{1}{n+1} \cdot \frac{1}{\binom{n}{k}}} \\ &= \frac{\frac{1}{n+2} \cdot \frac{1}{\binom{n+1}{k+1}}}{\frac{1}{n+1} \cdot \frac{1}{\binom{n}{k}}} \\ &= \frac{k+1}{n+2} \end{aligned}$$

□

A.2 Proof of claim 8.4

Claim. *The optimal price is 1/3.*

Proof. The proof is based on the claim that there is a significant risk of a negative cascade for any price over 1/3:

- If $p > 2/3$ then no agent will buy the the firm.
- If $p \in (3/5, 2/3]$, then if at least one of the first two agents does not buy, the expected value of the next agent, even after receiving an H is bounded by $\frac{2+1}{3+2} = 3/5$, so with probability

$$1 - \Pr[\text{the first two agents buy}] = 1 - \frac{1}{2+1} = \frac{2}{3}$$

the entrepreneur sells nothing, and thus the expected revenue is less than the promised 1/3.

- If $p \in (1/2, 3/5]$, then $p > 3/6$ and thus a negative cascade occurs for the histories: LL, HLL, LHL , whose total probability is:

$$\frac{1}{2+1} + \frac{1}{3+1} \cdot \frac{1}{\binom{3}{1}} + \frac{1}{3+1} \cdot \frac{1}{\binom{3}{1}} = \frac{1}{2}$$

Therefore the expected revenue is bounded by $\frac{1}{2} \cdot \frac{3}{5} = \frac{3}{10} < \frac{1}{3}$.

- If $p \in (2/5, 1/2]$, then a negative cascade occurs for LL with probability $\frac{1}{3}$ and also for $LHLLL$ with probability > 0 . The expected revenue is thus strictly lower than $\frac{2}{3} \cdot \frac{1}{2} = \frac{1}{3}$.
- Finally, for $p \in (1/3, 2/5]$, a negative cascade results from LLL - with probability $\frac{1}{4}$, and we can bound the expected revenue by $\frac{3}{4} \cdot \frac{2}{5} = \frac{3}{10} < \frac{1}{3}$

□