Assignment 1 - Geometric Optimization (0368-4144)

Due: November 9, 2009

Problem 1

Distance selection in the L_∞-norm. Let P be a set of n points in the plane (in general position), and let $1 \le k \le \binom{n}{2}$ be a given parameter. Find the k-th smallest L_∞-distance among the pairs of points of P. (Recall that the L_∞-distance between $a = (a_1, a_2)$ and $b = (b_1, b_2)$ is $d_\infty(a, b) = \max\{|a_1 - b_1|, |a_2 - b_2|\}$.)

Use parametric searching, and give details concerning the implementation of the generic procedure (parallel algorithm, critical values, etc.) What is the running time of the algorithm?

(**Hint:** One way of implementing the decision procedure is to use orthogonal range searching.)

Problem 2

Rectilinear 2-center: Let P be a set of n points in the plane. Using parametric searching, find two axis-parallel squares of smallest possible (equal) size, whose union contains P.

Give details concerning the decision procedure, generic implementation, etc. What is the running time of the algorithm? (**Note:** This is not the optimal way to solve the problem.)

Problem 3

Let $x_1, \ldots, x_n$ be n distinct real numbers. We choose a random sample R of them, so that each number is chosen independently with success probability $p = r/n$ (so the expected size of the sample is r).

(a) Show that, with high probability, every interval between two consecutive elements of R, as well as the interval preceding the smallest one and the interval succeeding the largest one, contains at most $\frac{cn}{r} \log r$ of the original numbers x_i. (The probability increases with the constant c.)

(b) Consider the slope selection problem, for a set L of n lines and paremeter k. Choose a random sample R of the $\binom{n}{2}$ vertices of the arrangement of L, as above, with $r = cn \log n$. Derive, using (a), a randomized algorithm that solves the slope selection problem in expected time $O(n \log^2 n)$ (without parametric searching).

Problem 4

1-center with outliers. Let S be a set of n points in the plane, and $k < n$ an integer. Find the smallest disk that contains all but k points of S. Describe briefly an algorithm, based on parametric searching, that runs in roughly quadratic time.

Describe in more detail the parallel implementation of the decision procedure, which can be reduced to the following: Given n equal disks in the plane, find a point that lies in the maximum number of disks. (**Hints:** (i) Line sweeping is hard to parallelize, so avoid it. (ii) Show (and use) the fact that it suffices to look for the deepest point on the boundaries of the given disks.)

Problem 5

Let A be a set of n real numbers and let B be another set of n real numbers. Define $A+B$ to be the set $\{x+y \mid x \in A,\, y \in B\}$. (Assume for simplicity that all sums are distinct.) Given a parameter $1 \leq k \leq n^2$, find the k-th smallest element of $A+B$. (**Hint:** Use monotone matrix searching.)