Assignment 3 - Geometric Optimization (0368-4144)

Due: December 21, 2009

Problem 1

Use $(1/r)$-cuttings to solve the following problem: Let D be a set of n disks in the plane. Preprocess D into a data structure so that, given a query point q we can efficiently (in $O(\log n)$ time) compute the number of disks of D which contain q. What is the storage used by the structure?

Problem 2

Explain the behavior of the subexponential LP algorithm on the following set H of $2d$ constraints in $\mathbb{R}^d$ (which define a hypercube):

$$1 \le x_i \le 2, \qquad i = 1, \ldots, d,$$

where we wish to minimize the objective function $\frac{1}{3}x_1 + \frac{1}{3^2}x_2 + \cdots + \frac{1}{3^d}x_d$. (In addition, we have the set H^+ of limiting constraints $x_i \ge 0$, for $i = 1, \ldots, d$.)

Address issues like: What is the form of a basis B? How does the basis recomputation routine $\texttt{basis}(B, h)$ work? What is the hidden dimension of a basis B within a set $G \subseteq H$ of constraints? Explain explicitly why the hidden dimension always becomes smaller during the execution of the algorithm. Describe the derivation of a recurrence for the expected running time in this special case.

Problem 3

Explain in detail why the following problems are LP-type: Define the set H of constraints, the set $\mathcal{W}$ of values, the function w from subsets of constraints to $\mathcal{W}$, explain why w is monotone and local, and show an upper bound on the maximum size of a basis (the combinatorial dimension of the problem). Discuss whether the problem is basis-regular or not.

(a) Given two convex polytopes P, Q in $\mathbb{R}^d$, each specified as the convex hull of n points, find $p \in P$, $q \in Q$ that minimize the distance $d(p, q)$. (p and q need not be vertices of the polytopes!)

(b) Given a set P of n points on a line, find four intervals of the same length w whose union contains P, for which w is as small as possible. (Show, as a preliminary step, that P can be covered by four intervals of length w if and only if every 5 points of P can be covered by four such intervals (not necessarily the same).)

Problem 4

Explain why the following problems are not LP-type (with finite combinatorial dimension that does not depend on n):

(a) Given n points on the line, find the smallest distance between a pair of them.

(b) Given a set P of n points in the plane, find two equal disks of the smallest possible radius whose union covers P. (Explain why the radius of the disks is not a good value-function.) (**Hint:** Consider m points equally placed on some circle. Is such a set a basis?)