# Assignment 5 - Geometric Optimization (0368-4144)

Due: January 25, 2010

## Problem 1

**Smallest enclosing cylinder in the $L_\infty$-norm.** Let $P$ be a set of $n$ points in 3-space. Find a line $\ell$ such that the maximum $L_\infty$ distance from the points of $P$ to $\ell$ is minimized. (**Hint:** In fact, this is an LP-type problem!)

## Problem 2

**Smallest enclosing cylinder.** Let $P$ be a set of $n$ points in 3-space. Find a line $\ell$ that passes through the origin such that the maximum Euclidean distance from the points of $P$ to $\ell$ is minimized.

## Problem 3

**Diameter in four dimensions.** Give an exact, sub-quadratic algorithm for computing the diameter of a set $P$ of $n$ points in four dimensions. (**Hint:** Here, the ball intersection procedure can be simplified by lifting the points to the paraboloid in five dimensions, so that balls become halfspaces, and the problem becomes that of determining whether all points lie below all hyperplanes. See standard textbooks if you are not familiar with this standard transformation. Now use cuttings.)

## Problem 4

Let $P$ be a set of $n$ points in three dimensions. Give a $(1+\varepsilon)$-approximation LTAS algorithm for computing a box (not necessarily axis parallel) enclosing $P$, such that the sum of its edges is minimal.

Describe briefly an exact algorithm for solving the problem. It can be inefficient but it should be polynomial in $n$. Then use a grid approximation combined with such an algorithm to solve the approximation problem. (Any other solution is of course also acceptable, but then still describe an exact solution, as above.)