Assignment 5 - Geometric Optimization (0368-4144)

Due: Before the exam, in my mailbox

Problem 1

Smallest enclosing cylinder in the L_∞-norm. Let P be a set of n points in 3-space. Find a line ℓ such that the maximum L_∞ distance from the points of P to ℓ is minimized. (**Hint:** In fact, this is an LP-type problem!)

Problem 2

Smallest enclosing cylinder. Let P be a set of n points in 3-space. Find a line ℓ that *passes through the origin* such that the maximum Euclidean distance from the points of P to ℓ is minimized.

(a) Give an exact algorithm: Formulate the problem in an LP-style, as minimizing an objective function subject to constraints. Then linearize the constraints, compute the feasible region and search for a minimum of the objective function.

(b) Let Q be an ε-kernel of P. Show that the radius of the smallest enclosing cylinder for Q (with its axis through the origin, as above) is a good approximation for the radius for P.

What are the consequences for an approximate algorithm?

Problem 3

Approximate smallest enclosing ball. Let P be a set of n points in $\mathbb{R}^d$. Show that an ε-kernel $Q \subset P$ can be used to approximate the radius of the smallest enclosing ball of P, in the sense that the radius of the smallest enclosing ball of Q is at least $1 - O(\varepsilon)$ times that for P.