about the probability distribution of the input points. This problem brings us into the province of stochastic geometry, which deals with the properties of random geometric objects and is an essential tool for dealing with expected-time analysis.[1] We would like to be able to say, "Given N points chosen uniformly in the plane . . . ," but technical difficulties make this impossible—elements can be chosen uniformly only from a set of bounded Lebesgue measure [Kendall–Moran (1963)] so we are forced to specify a particular figure from which the points are to be selected. Fortunately, the problem of calculating $E(h)$ has received a good deal of attention in the statistical literature, and we quote below a number of theorems that will be relevant to the analysis of several geometric algorithms.

Theorem 4.1 [Rényi–Sulanke (1963)]. *If N points are chosen uniformly and independently at random from a plane convex* r-gon, *then as* $N \to \infty$,

$$E(h) = \left(\frac{2r}{3}\right)(\gamma + \log_e N) + O(1), \tag{4.1}$$

where γ *denotes Euler's constant.*

Theorem 4.2 [Raynaud (1970)]. *If N points are chosen uniformly and independently at random from the interior of a k-dimensional hypersphere, then as* $N \to \infty$, $E(f)$, *the expected number of facets of the convex hull, is given asymptotically by*

$$E(f) = O(N^{(k-1)/(k+1)}). \tag{4.2}$$

This implies that

$E(h) = O(N^{1/3})$ *for N points chosen uniformly in a circle, and*

$E(h) = O(N^{1/2})$ *for N points chosen uniformly in a sphere.*

Theorem 4.3 [Raynaud (1970)]. *If N points are chosen independently from a k-dimensional normal distribution, then as* $N \to \infty$ *the asymptotic behavior of* $E(h)$ *is given by*

$$E(h) = O((\log N)^{(k-1)/2}). \tag{4.3}$$

Theorem 4.4 [Bentley–Kung–Schkolnick–Thompson (1978)]. *If N points in k dimensions have their components chosen independently from any set of continuous distributions (possibly different for each component), then*

$$E(h) = O((\log N)^{k-1}). \tag{4.4}$$

Many distributions satisfy the conditions of this theorem, including the uniform distribution over a hypercube.

[1] Consult [Santalò (1976)] for a monumental compilation of results in this field.