

Highly Accurate Numerical Methods for Chemotaxis and Related Models

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In this talk, I will present a new family of high-order finite volume method for the Keller-Segel chemotaxis system and several related models. In its simplest form, the chemotaxis model is described by a system of nonlinear PDEs: a convection-diffusion equation for the cell density coupled with a reaction-diffusion equation for the chemoattractant concentration. It is well known that solutions of such systems may develop spiky structures or even blow up in finite time provided the total number of cells exceeds a certain threshold. This makes development of numerical methods for chemotaxis systems extremely delicate and challenging task.

The first step in the derivation of the new method is made by adding an equation for the chemoattractant concentration, gradient to the original system. The convective part of the resulting system is then of a mixed hyperbolic-elliptic type and therefore straightforward numerical methods for the studied system may be unstable. We design a new method, which is based on the application of Godunov-type central-upwind schemes, originally developed for hyperbolic systems of conservation laws, to the chemotaxis systems. We prove that the proposed high-order central-upwind schemes are positivity preserving, which is a very important stability property of the method.

We apply the new schemes to a number of two-dimensional problems including the most commonly used Keller-Segel chemotaxis model and its modern extensions as well as to a haptotaxis system modeling tumor invasion into surrounding healthy tissue. If time permits, I will also discuss a model describing the collective behavior of a suspension of oxygen-driven bacteria in an aquatic fluid.