

Tumour angiogenesis model with variable diffusion coefficient

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Abstract. In adults normal physiological role of angiogenesis is restricted to wound healing, the menstrual cycle and pregnancy. Unfortunately, it is also essential for the successful growth and development of solid tumours. After reaching avascular dormant state tumour can grow further only by inducing host tissue to sprout capillary tubes which migrate towards and ultimately penetrate the tumour, providing it with a circulating blood supply and, therefore, an additional source of nutrients. On the basis of idea that carrying capacity for any solid tumour depends on its vessel density Hahnfeldt et. al created mathematical model of tumour growth under angiogenic signalling. Dr. Judith Folkman who discovered process of tumour angiogenesis proposed that tumour can be treated by influencing on that process. On the basis of the Hahnfeldt et. al model some protocols of antiangiogenic treatment was proposed.

Unfortunately recent studies show that tumour angiogenesis is highly pathological. Long lasting over-expression of pro-angiogenic factors (like VEGF) provides impairment and malfunction of newly formed vessels. We propose the model of vascular tumour growth, which generalises the well known model formulated by Hahnfeldt et al. in 1999. It is based on the same idea that the carrying capacity for any solid tumour depends on its vessel density but it also incorporates vasculature quality which may be lost during the conduction of angiogenesis as recognised by Jain in 2005. In the model we assume that the loss of vessel quality affects the diffusion coefficient inside the tumour. More precisely, we assume that in diffusion-consumption equation

$$\overline{D}^2 \nabla^2 n - cn + s = \frac{\partial n}{\partial t},$$

which was used by Hahnfeldt et.al. to ascertain nature of carrying capacity dynamics, diffusion coefficient $\overline{D}^2 = D(t, \mathbf{x})$ is no longer a constant value. In the first approximation we assume that $D(t, \mathbf{x})$ is an average value of diffusion coefficients in the whole tumour region, that is it is only dependent on time. In that case we derive the model consisted of three ordinary differential equations describing tumours' volume, vessels' volume and diffusion coefficient value. We analyse its basic mathematical properties and present some numerical simulations.

Secondly we assume that diffusion coefficient depends also on the distance from tumour center, that is $\overline{D}^2 = D(t, r)$ is an average value of diffusion coefficients over a sphere of radius r . In that case we perform the series of numerical simulations to investigate levels of stimulators' and inhibitors' secretion.