# Investigation of passive arterial mechanical response by high order finite element methods

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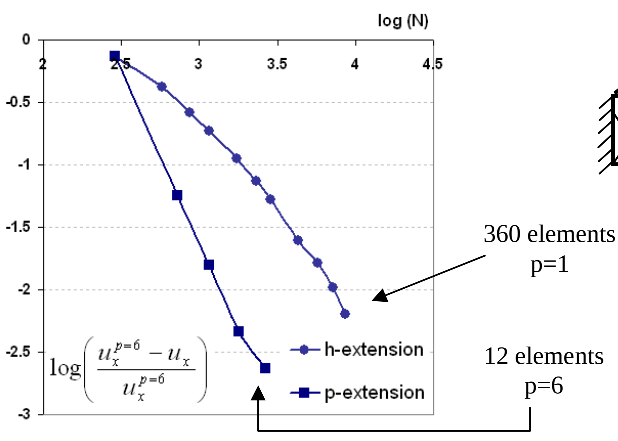
Proper mathematical formulation for predicting the mechanical response of arterial walls is of cardinal importance in understanding the development of arterial diseases and their treatment. The complicated internal structure of the artery (being inhomogeneous and orthotropic) combined with the large deformations under physiological conditions necessitate a non-linear analysis and a complex constitutive model. As a result, special numerical methods such as high-order finite elements are to be developed to address such problems.

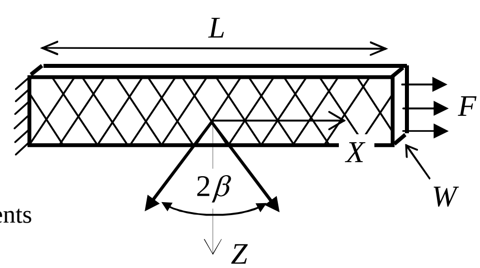
Using the hyper-elastic theory, each of the mechanically significant arterial layers (media, adventitia), is modeled as a homogenous isotropic nearly incompressible matrix with two families of collagen fibers [1] (arranged in a helix like formation with an angle $\pm\beta$ between the fiber and the circumferential direction).

$$\psi(I_{c1}, I_{c3}, I_{c4}, I_{c6}) = \underbrace{\frac{\mu}{2}\left(I_{c1} I_{c3}^{-\frac{1}{3}} - 3\right) + \frac{\kappa}{2}\left(\sqrt{I_{c3}} - 1\right)^2}_{isotropic} + \underbrace{(k_1 / 2k_2) \sum_{i=4,6} \left\{ \exp\left[ k_2 \left( I_{ci} I_{c3}^{-\frac{1}{3}} - 1 \right)^2 - 1 \right] \right\}}_{anisotropic}$$

where $\mu, \kappa$ are the shear and bulk modulus of the matrix, $k_1, k_2$ represent the fiber stiffness. $I_{c1,c3}$ are invariants of the right Cauchy-Green deformation tensor while $I_{c4,c6}$ correspond to stretch in the fiber directions.

In this talk the mathematical formulation is presented and its discretization by p-FE methods is formulated. Verification of the numerical schemes are performed by comparing these to analytical solutions of a set of simplified "benchmark" problems. Several problems are introduced that discuss the effect of compressibility, fiber orientation and layer thickness ratio. For example, in the figure bellow we compare the convergence of the traditional h-version compared to the p-version of the FEM. The relative error in maximum displacement for a nearly incompressible $\kappa \approx 10000$ arterial adventitial thin strip $L/W = 60$ clamped at one end with initial fiber orientation of $\beta = \pm 40^0$ is plotted against increasing degrees of freedom N.




*[1] Holzapfel, G.A. and Gasser, T.C. and Ogden, R.W.* **2000**, *"A new constitutive framework for arterial wall mechanics and a comparative study of material models, J. Elast (61). pp. 1-48*