

Invariant Feature Detection for Abelian and non-Abelian Lie Transformation Groups

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In many medical imaging situations one has to recognize regions or textures that undergo a transformation due to geometrical changes in acquisition. These geometrical transformations are best modeled by Lie groups. We present a complete system of functionally independent invariant feature detectors for a general Abelian Lie transformation group. We provide also such an invariant for the $SL(2, \mathbf{R})$ Lie transformation group. This group is non-Abelian and may be decomposed via the Iwasawa decomposition into meaningful transformations on images. The features in this framework are extracted via inner products between a set of measuring functions and an image. The (infinitesimal) Lie transformation is generated by a differential operator with respect to these features and the appropriate group representations. Invariance yields a system of PDEs. The solutions to these PDE systems are functions of the features, and do not change under common image transformations. Hence, they are called image invariants or invariant feature detectors. We construct different kinds of image invariants for the Abelian and the non-Abelian transformation groups. Finally, we test these invariants by some measurements on images.