2010 IEEE International Workshop on Machine Learning for Signal Processing (MLSP 2010)
August 29 – September 1, 2010, Kittilä, Finland

SNR-DEPENDENT FILTERING FOR TIME OF ARRIVAL ESTIMATION IN HIGH NOISE

Alexander Apartsin[1], Leon N Cooper[2], Nathan Intrator[1, 2]

[1] Blavatnik School, Of Computer Science, Tel-Aviv University, [2] Institute for Brain and Neural Systems and Physics Department, Brown University

ABSTRACT

Time of Arrival (ToA) estimation is a cornerstone of many of the remote sensing applications including radar, sonar, and reflective seismology. The conventional Matched Filter Maximum Likelihood (MFML) ToA estimator suffers from rapid deterioration in the accuracy as Signal to Noise Ratio (SNR) falls below certain threshold value. In this paper we suggest an alternative method for ToA estimation based on the fusion of measurements from biased estimators which are obtained using a pair of unmatched filters. Suboptimal but not perfectly correlated estimators are combined together to produce a robust estimator for ToA estimation in high noise. The unmatched filters pair is parameterized by a single parameter (phase shift) which is selected based on estimated SNR level.

Index Terms-Time of Arrival, Threshold Effect

I. INTRODUCTION

In remote sensing applications such as radar or sonar, the common scenario starts by a transmitter sending out a pulse waveform $\boldsymbol{s(t)}$ **.** The pulse is reflected from a target and it is picked up by a receiver at time $\mathbf{t_0}$ **.** The estimated two-way travel time (lag) can be used to calculate distance to the target assuming the speed of the pulse propagation in the medium is known.
The signal recorded at the receiver might be represented as

$$\boldsymbol{u(t) = c * s(t - t_o) + n(t)}$$

where $\boldsymbol{n(t)}$ is Additive White Gaussian Noise(AWGN) which corrupts the signal. The $\boldsymbol{c} < 1$ factor is used to account for all non-free space propagation losses (e.g. attenuation of the signal in the medium). We are interested in estimating the Time of Arrival (ToA) parameter $\mathbf{t_0}$ under the assumption that noise $\boldsymbol{n}$ is large relative to **c*s(t)**. The analysis of the performance of different time-of-arrival estimation methods is essential for Radar, Sonar and other remote sensing applications. Rather than compute the exact error of a specific estimator, it is often more convenient to lower-bound the error of any estimators for a given problem. The conventional Matched Filter Maximum Likelihood (MFML) estimator is considered efficient as it asymptotically attains the Cramer-Rao Bound (CRB) under sufficiently high SNR conditions **[10]**. However, under lower SNR levels, the Cramer-Rao Bound appears to be over-optimistic and a more tight forms of bound are required if the level of noise is high. The Barankin Bound **[9]** and associated Barankin Theory provide tools for constructing useful bounds for mean error of an estimator under low SNR. Although in its general form the Barankin bound depends on the estimated parameter and therefore can't be easily computed, it is able to account for well-known threshold phenomena in the estimation of the time-of-arrival parameter.

This work was supported in part by the U.S. Army Research Office

According to Woodward who studied the threshold effect back in 1953 **[6],** it is "one of the most interesting features of radar theory". It appears that when SNR at a receiver falls below certain threshold value, the mean square error of the estimation is rapidly increasing causing dramatic drop in sensing accuracy. A receiver operating with SNR above this threshold value is said to be in a coherent state. The MFML estimator is usually used for the coherent receiver. For the SNR levels substantially below the threshold value, a receiver said to be noncoherent with the assumption that most of the information about the pulse carrier phase is lost due to the noise. For in-between levels of SNR, a receiver is said to be a semi-coherent receiver, balancing between coherent and noncoherent states.

In this paper we describe a robust single pulse ToA estimation method for semi-coherent receiver. We show how to construct a pair of suboptimal and biased estimators, using phase-shifted versions of source waveform as unmatched filters. The outcomes of an estimator pair are fused together into a single ToA estimator which outperforms MFML estimator for a range of low SNR levels. We introduce an SNR-dependent ToA estimator by selecting phase-shift value according to anticipated SNR at a receiver.

II. MAXIMUM LIKELIHOOD MATCHED FILTER ESTIMATOR

The standard method for ToA estimation employs Matched Filter (MF) applied to the received signal. The Matched Filter maximizes peak signal to mean noise ratio

978-1-4244-7877-4/10/$26.00

[1], [7], making its output suitable for the Maximum Likelihood (ML) estimator of the ToA. The Matched Filter Maximum Likelihood (MFML) estimator of ToA is obtained by taking the position of the global maximum in the output of the Matched Filter (MF). The output of the Matched Filter can be expressed as a correlation of the signal with the pulse waveform:

$$\boldsymbol{y}(\boldsymbol{t}) = \boldsymbol{u}(\boldsymbol{t}) \circ \boldsymbol{s}(\boldsymbol{t}) = \boldsymbol{g}(\boldsymbol{t}) + \boldsymbol{h}(\boldsymbol{t})$$

Where $\boldsymbol{g}(\boldsymbol{t})$ is scaled and shifted version of the pulse's autocorrelation function and $\boldsymbol{h}(\boldsymbol{t})$ is filtered noise. A typical Gaussian-modulated sinusoidal pulse and its autocorrelation function are shown in **Figure 1**.

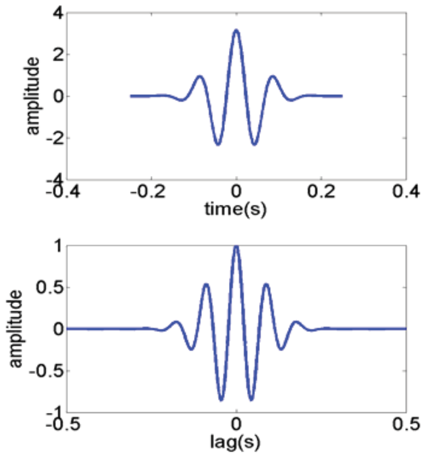

Figure 1: Gaussian modulated sinusoidal pulse (top) and its autocorrelation function ***y(t)***.

In the absence of noise, the maximum value of $\boldsymbol{y}(\boldsymbol{t})$ is achieved at $\boldsymbol{t} = \boldsymbol{t_o}$. As the level of noise increases, the filtered noise $\boldsymbol{h}(\boldsymbol{t})$ may cause a slight shift in the location of the peak of $\boldsymbol{y}(\boldsymbol{t})$**.** However, at the high noise levels, a location around one of the side lobes of $\boldsymbol{g}(\boldsymbol{t})$ may occasionally become the global maximum of $\boldsymbol{y}(\boldsymbol{t})$.

A side lobe of autocorrelation function mistakenly taken as its global maximum is a major reason behind deterioration in accuracy of MFML estimator known as threshold effect **[6]**. The threshold effect occurs as soon as Signal to Noise Ratio (SNR) $\boldsymbol{R}$ falls below the level that is given approximately by

$$\boldsymbol{R} \sim \mathbf{2}\log(\boldsymbol{F} * \boldsymbol{R} * \boldsymbol{B})$$

where $\boldsymbol{F}$ is the detection interval and $\boldsymbol{B}$ is the signal bandwidth.

The threshold effect manifests itself in rapid increase in the Root Mean Square Error (RMSE) of the MFML estimator as shown in the **Figure 2**. In semi-coherent state, the posteriori distribution of the possible lag locations becomes multimodal (**Figure 3**) because of the significant height of autocorrelation function's side lobes. The height of the side lobes of the autocorrelation function is affected by the pulse bandwidth. Therefore, the threshold effect is considerable for low-frequency narrowband pulses. Under these circumstances, the Maximal Likelihood Matched Filter behaves poorly.

Many of the commonly used source waveforms have side lobes in their autocorrelation function (e.g. [13]). Therefore, although the effectiveness of the proposed method is demonstrated using Gaussian-modulated sinusoidal pulse, the method can be applied to other source waveforms as well.

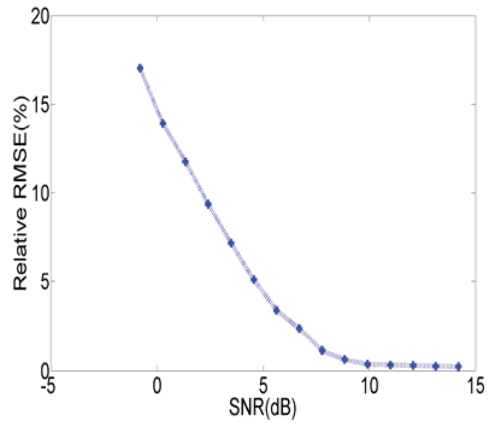

Figure 2: The MFML estimator threshold effect. The error increases rapidly as SNR falls below a threshold.

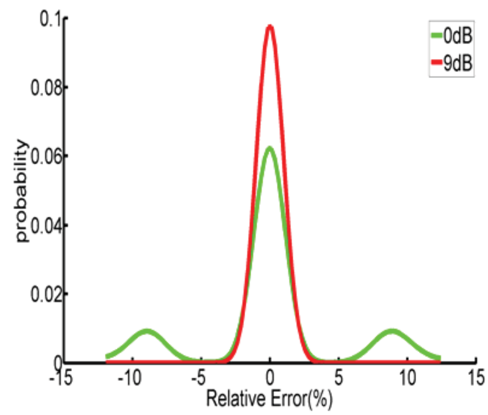

Figure 3: The probability density function for MFML estimator error. There are significant local maxima under low SNR

III. UNMATCHED FILTER MAXIMUM LIKELIHOOD ESTIMATOR

Given an arbitrary pulse waveforms $\boldsymbol{s}(\boldsymbol{t})$**,** we construct a pair of Phase Shifted Unmatched (PSU) filters $\boldsymbol{f}_{\boldsymbol{\varphi}}^{+}(\boldsymbol{t})$ and $\boldsymbol{f}_{\boldsymbol{\varphi}}^{-}(\boldsymbol{t})$ by shifting the phase of each pulse by $+\boldsymbol{\varphi}$ and $-\boldsymbol{\varphi}$ respectively.

A Gaussian-modulated sinusoidal pulse and its PSU filter pair generated using $\boldsymbol{\varphi} = \frac{\boldsymbol{\pi}}{\mathbf{2}}$ are shown in **Figure 4.** The cross correlation of the signal $\boldsymbol{u}(\boldsymbol{t})$ and a PSU pair's filter can be expressed as:

$$\mathbf{y}_{\boldsymbol{\varphi}}^{\pm}(\mathbf{t}) = \boldsymbol{u}(\boldsymbol{t}) \circ \mathbf{f}_{\boldsymbol{\varphi}}^{\pm}(\mathbf{t}) = \mathbf{g}_{\boldsymbol{\varphi}}^{\pm}(\boldsymbol{t}) + \mathbf{h}_{\boldsymbol{\varphi}}^{\pm}(\boldsymbol{t})$$

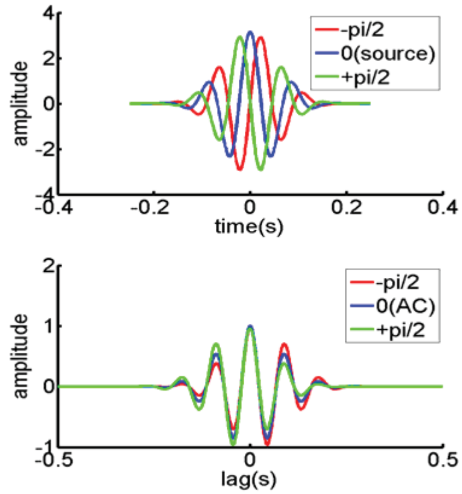

Figure 4: Phase shifted pulses (top) and their cross correlation functions (bottom). Note asymmetric shape of side lobes

The Unmatched Filter Maximum Likelihood (UFML) estimators t_φ^- and t_φ^+ corresponding to a PSU pair can be defined as:

$$t_\varphi^\pm = argmax\left(\mathrm{y}_\varphi^\pm(\mathrm{t})\right) = argmax(u(t) \circ f_\varphi^\pm(t))$$

The side lobes of the cross-correlation function $\mathbf{g}_\varphi^\pm(\mathbf{x}) = \mathbf{s}(\mathbf{t}) \circ \mathbf{f}_\varphi^\pm(\mathbf{t})$ have unequal heights, making the UFML estimators biased toward the higher side lobe as shown in **Figure 5**.

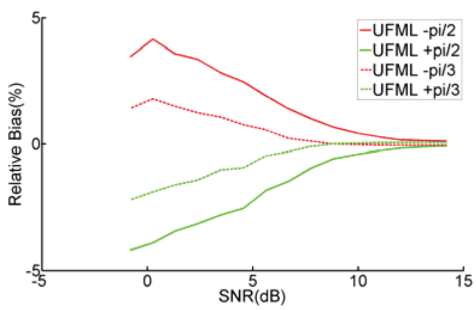

Figure 5: Bias of UFML estimator pair. Unmatched filter pair produces biased estimator pair with bias of the same value but opposite sign.

The bias of the two UFML estimators has equal absolute value but opposite sign due to symmetry in the heights and position of the cross-correlation side lobes. As SNR is increased, the bias decreases since the position of the cross-correlation maximum is less affected by the noise. Note that autocorrelation and PSU filter cross-correlation produce signals of the same power, however application of unmatched filter produces lower peak signal-to-mean-noise ratio as compared to matched filter.

The Root Mean Square Error (RMSE) of a single UFML estimator is higher as compared to the RMSE of MFML as shown in **Figure 6**. However, the UFML estimators corresponding to a PSU filter pair are not perfectly correlated as can be seen in **Figure 7**.

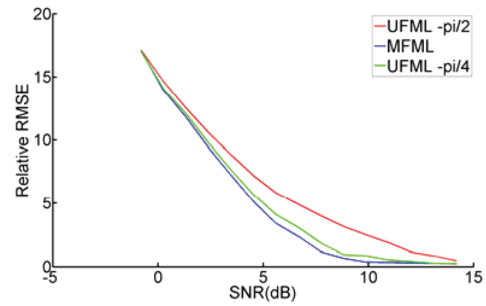

Figure 6: Relative RMSE of UMFL estimators. Each UMFL estimator produces suboptimal error.

Therefore we can define a new estimator by averaging results from a pair of UFML:

$$t_\varphi = \frac{t_\varphi^- + t_\varphi^+}{2}$$

At low SNR levels, the resulting Average of UFML (AoUFML) estimator has lower RMSE as compared to MFML (**Figure 8**).

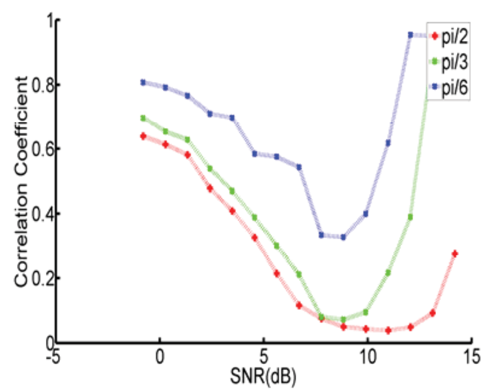

Figure 7: Cross-Correlation coefficient of UFML pair. At larger SNR the main peak becomes dominant; therefore correlation coefficient is close to one. For very small SNR, the estimate is mostly affected by the noise shape thus resulting in correlated estimators. For intermediate values of SNR the correlation is small.

The AoUFML estimator outperforms MFML estimator at SNR levels corresponding to semi-coherent receiver state. At higher SNR levels, the effect of side lobes is insignificant therefore the shape of the main peak of cross-correlation function have critical impact on the estimator's RMSE. Since an unmatched filter produces smaller peak signal-to-mean-noise ratio and the UFML pair is almost perfectly correlated at higher SNR levels, the MFML estimator outperform the AoUFML estimator t_φ at coherent receiver state.

The cross-over points between AoUFML and MFML RMSE curves can be controlled by choosing appropriate phase shift parameter $\boldsymbol{\varphi}$ as described below.

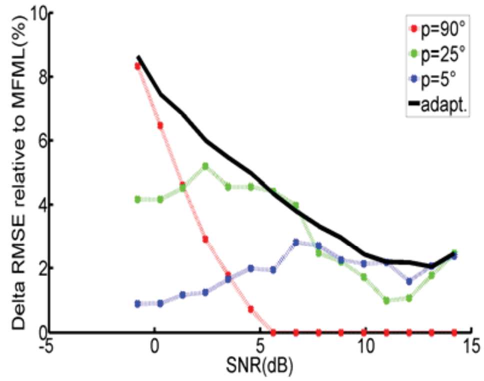

Figure 8: RMSE improvement by fixed and adaptive phase AoUFML estimators. For each SNR there is the best performing value of a phase shift (color lines). The black line shows error for adaptive selection of phase-shift value

IV. PHASE SELECTION

There are two factors affecting the selection of the value $\boldsymbol{\varphi}$ for generating PSU filter pair. It is desirable to minimize the correlation between the values of the additive noise $\mathbf{h}_{\boldsymbol{\varphi}}^{+}(\boldsymbol{t})$ and $\mathbf{h}_{\boldsymbol{\varphi}}^{-}(\boldsymbol{t})$, while keeping peak signal-to-mean-noise level of $\mathbf{y}_{\boldsymbol{\varphi}}^{\pm}(\mathbf{t})$ close to that of a Matched Filter output $\boldsymbol{y(t)}$.

Since the average noise level is not affected by PSU filtering, the peak SNR level depends on the height of the main of the peak of the cross-correlation function. The peak height is smaller with larger values of the phase shift $\boldsymbol{\varphi}$**,** meaning that $\boldsymbol{\varphi}$ should be decreased as the SNR level approaches the coherent range. On the other hand, the correlation between the noise phases decreases with larger phase, becoming zero when $\boldsymbol{\varphi} = \frac{\boldsymbol{\pi}}{\mathbf{2}}$, making the noise phases of $\mathbf{h}_{\boldsymbol{\varphi}}^{+}(\boldsymbol{t})$ and $\mathbf{h}_{\boldsymbol{\varphi}}^{-}(\boldsymbol{t})$ orthogonal.

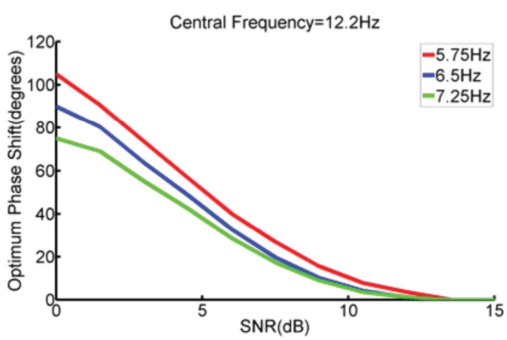

Figure 9: The optimum value of phase shift as a function of SNR for different values of pulse bandwidth. For larger bandwidth, the optimal phase shift value is smaller. The optimal phase shift value is learned from the simulation data.

In general, the optimal value of $\boldsymbol{\varphi}$ from the interval $\left[\mathbf{0}:\frac{\boldsymbol{\pi}}{\mathbf{2}}\right]$ can be selected according to the estimated levels of noise present in the signal. This principle is illustrated in **Figure 8** which shows the RMSE curves for several values of $\boldsymbol{\varphi}$ and the RMSE curve corresponding do adaptively selected phase shift value $\boldsymbol{\varphi}$**.** Using curves presented in **Figure 9** and **Figure 10,** the optimal value of the phase-shift for a given pulse can be selected based on estimated level of SNR (the review of SNR estimation techniques can be found in **[8,14]**). The optimal value of the phase shift value used in AoUFML estimator depends on the pulse central frequency and the pulse bandwidth. A pulse with larger bandwidth has a narrower envelope of the auto-correlation function. Therefore smaller values of phase shift produce larger changes in the height of a side lobe. As a result, the optimal value for AoUFML phase shift is smaller for pulses with larger bandwidth (**Figure 9**).

In a similar manner, as the central frequency of the pulse is increased, the peaks of autocorrelation function become more closely spaced. Therefore larger values of phase shift can be used in UFML in order to produce significant difference in side-lobes height (**Figure 10**).

Given the above consideration, the optimal value of a phase shift can be learned (tabulated) from simulated data produced for a range of SNR and phase shift values. Based on the simulation results, the optimal filter pair for the measurement can be selected after the anticipated SNR level is estimated using techniques described in **[8,14].**

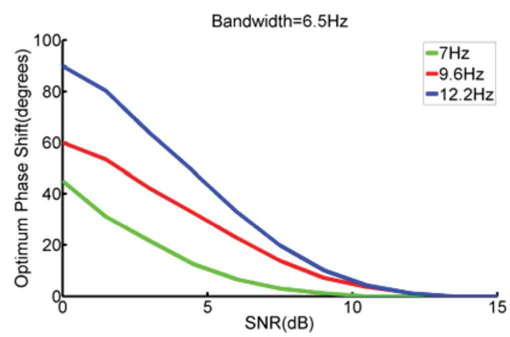

Figure 10: The optimum value of phase shift as a function of SNR for different values of pulse central frequency. For larger central frequency, the optimal value of a phase shift value is larger. The optimal phase shift value is learned from the simulation data.

V. CONCLUSIONS

We showed that using Phase Shifted Unmatched (PSU) filters, a pair of Unmatched Filter Maximum Likelihood (UFML) estimators can be applied to obtain biased Time of Arrival estimators. In semi-coherent receiver state, the UFML estimators are not perfectly correlated and, therefore, can be combined together into estimator that outperforms conventional Matched Filter Maximum Likelihood estimator. There is an optimal phase shift level that produce lowest RMSE for given SNR. Therefore a phase shift level can be selected adaptively according to estimated SNR level in order to achieve best accuracy in a semi coherent state. Since the proposed estimator does not require special pulse waveform or additional pulses, it can be used during a post-processing phase in remote sensing applications operating under high noise.

VI. REFERENCES

[1] *D. Whalen, "Detection of Signals in Noise", Academic Press, 1995*

[2] L. R. Varshney and D. Thomas, "*Sidelobe reduction for matched filter range processing*", Proceedings of IEEE Radar Conference, 2003, pp446-451

[3] GP Succi, G Prado, R Gampert and TK Pedersen, "*Problems in Seismic detection and Tracking*", Proceedings of SPIE 2000, vol. 4040, pp165-173

[4] Tali Kimchi, Moshe Reshef**,** Joseph Terkel, "*Evidence for the use of reflected self-generated seismic waves for spatial orientation in a blind subterranean mammal*", Journal of Experimental Biology 2005, vol208, pp647-658

[5] L. Yu, N. Neretti, N. Intrator, "*Multiple ping sonar accuracy improvement using robust motion estimation and ping fusion*", JASA 2006, Volume 119, Issue 4, pp2106-213

[6] P. Woodward, "*Probability and Information Theory, with Applications to Radar*", McGraw-Hill, 1953

[7] Steven M. Kay, "*Fundamental of Statistical Signal Processing*", Prentice Hall, 1993

[8] D.R. Pauluzzi, N.C. Beaulieu, "*A comparison of SNR estimation techniques for the AWGN channel*", IEEE Transaction on Communications 2000,Volume 48, Issue 10, pp1681-1691

[9] E.W. Barankin, "*Locally best unbiased estimates*" Ann Math. Statist., 20:477-501,1949

[10] Harry L. Van Treese, "*Detection, Estimation and Modulation Theory*", John Wiley & Sons, Inc. New York 1968

[11] M.I. Skolnik, "*Introduction to Radar Systems*", McGraw-Hill Book Company 1962

[12] A.Zeira and P.M. Schultheiss, "*Realizable lower bounds for time delay estimation-Part II: Threshold phenomena*" IEEE Trans. Signal Processing, May 1994, Volume42, Issue 5, pp1001-1007

[13] Harold Ryan, "*Ricker, Ormsby, Klauder, Butterworth-A choice of wavelets",* CSEG Recorder, Vol. 19 #7, Sept. 1994

[14] Hong Shu Liao, Lu Gan,Ping Wei *"A blind SNR estimation method for radar signals",*IET international RADAR conference 2009