Biosonar Model for Improved Range Accuracy in a Noisy Environment.

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Abstract

Using the theory of optimal receivers the range accuracy of echolocating systems can be expressed as a function of receiver bandwidth and signal-to-noise ratio through the well-known Woodward equation. That equation however was developed in the limit of very high signal-to-noise ratios, and assumes that the correct peak of the crosscorrelation function is known a-priori. Echolocating animals such as dolphins and bats have developed a highly specialized receiver to optimize echolocation to different environments and conditions. In particular, they use a set of filters with different center frequencies but overlapping bands. We show that this structure can help in improving accuracy in the case of relatively low signal-to-noise ratios when the ambiguity in the choice of the main peak of the crosscorrelation function cannot be avoided.

Introduction

The theory of optimal receivers studies the design of pulses and receivers to obtain optimal detection in the presence of noise. In this paper we analyze the role of multiple filters on the receiver side to improve detection accuracy with respect to a given wideband pulse design. In our analysis we have been inspired by the structure of pulses and receiver in echolocating animals such as bats and dolphins, as the capabilities of the their sonar are still significantly better that any man-made system.

Considerable work on the theoretical accuracy of range measurements has been done in the past, and the Woodward equation has been derived using different methods. A comprehensive description can be found in [1]. However, it appears that interest in the mathematical aspects of the derivation of that equation has faded [2], while its use has become a standard in the field. In this letter we show that the validity of this equation depends on various assumptions in particular the assumptions of very low signal-to-noise ratios (SNR), therefore it must be reexamined for the case of low SNR's. The theory of optimal receivers shows that the *matched filter* receiver maximizes the output peak-signal-to-mean-noise (power) ratio [2, 3], and is the optimum method for the detection of signals in noise. Information about the distance of the target is extracted by computing the time at which the crosscorrelation between the echo and a replica of the pulse is a

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maximum. This delay is converted into a distance by means of the sound velocity in the particular medium in consideration (e.g. water or air). This type of receiver is generally referred to as a *coherent* receiver.

The classical theory of optimal receivers describes the range accuracy of a sonar system via the well-known Woodward equation, which can be derived by using a variety of methods[1, 4-7]. However, all of them rely upon the common crucial assumption of a large SNR, which implies a priori knowledge of the location of the central lobe in the crosscorrelation function. For small SNR's, one of the parameters in the classical equation – i.e. the bandwidth – has to be modified, and the receiver is then called *semicoherent*. In this letter we show that the transition between the two types of behaviors occurs at different SNR's depending on characteristics of the pulses such as bandwidth and center frequency. With this observation, we devise a novel system based on an adaptive choice of the pulse; this can improve accuracy in the case of relatively low SNR, when ambiguity in the choice of the correct peak of the crosscorrelation function cannot be avoided. This method can be generalized to the case of a fixed broadband pulse. In this case, both pulse and echo can be passed through a set of filters with appropriate center frequencies and bandwidths, and crosscorrelation can be performed separately in each frequency band.

Woodward equation

If we define $\psi_p(t)$ to be the pulse sent by the sonar and $\psi_e(t)$ to be the echo coming from a target at a distance d, then $\psi_e(t) = \psi_p(t+\tau_0) + \eta(t)$, where $\tau_0 = 2d/c$, c is the sound velocity in the particular medium in consideration (e.g. water or air), and $\eta(t)$ is in general white noise. The crosscorrelation between pulse and echo can be expressed as

$$\psi_e \circ \psi_p(\tau) = \int_{-\infty}^{+\infty} \psi_e(t)\psi_p(t+\tau)\mathrm{d}t = \int_{-\infty}^{+\infty} \psi_e(t)\psi_e(t+\tau+\tau_0)\mathrm{d}t + \int_{-\infty}^{+\infty} \psi_e(t)\eta(t+\tau)\mathrm{d}t, \qquad (1)$$

where the first term in the sum is the autocorrelation function of the pulse centered at τ_0, and the second term is band limited white noise, with frequency limits defined by the spectrum of the pulse. In the absence of noise, only the first term survives, and the distance from the target can be computed from the delay in time corresponding to the maximum of the crosscorrelation function. When the noise level is sufficiently low, its effect is to jitter the position of the maximum around the true value of the delay τ_0. To a first approximation, the jitter can be related to the width of the central peak in the autocorrelation function, which is a function of the signal's bandwidth and center frequency (Figure 1A). By using a rigorous argument based on the concept of inverse probability due to Woodward [6], it is possible to demonstrate that the standard deviation of the location in time of the maximum around the true value τ_0 is $\sigma_c = (2\pi\, B_{RMS}\, d)^{-1}$. In this formula B_{RMS} is the root mean square (RMS) bandwidth of the pulse and is defined as $B_{RMS} = \left(\int_0^{+\infty} f^2 P_{SD}(f)\mathrm{d}f\right)^{1/2}$, where $P_{SD}(f)$ is the power spectral density of the pulse, and the signal-to-noise ratio $d = \sqrt{2E/N_0}$ is a function of the ratio between the total energy E of the echo (measured in Ws), and the spectral density N_0 of the noise (measured in W/Hz = Ws). In the case of uniform Gaussian noise with variance σ_N^2, the spectral density of the noise sampled at a rate f_s can be expressed as $N_0 = 2\sigma_N^2 / f_s$. The signal-to-noise ratio is usually expressed in dB as $\mathrm{SNR_{dB}} = 20\log_{10} d$. Notice that the RMS bandwidth can be written as $B_{RMS}^2 = B_{CRMS}^2 + f_c^2$, where $f_c = \int_0^{+\infty} f \cdot P_{SD}(f)\mathrm{d}f$ is the center frequency of the signal, and $B_{CRMS} = \left(\int_0^{+\infty}(f - f_c)^2 P_{SD}(f)\mathrm{d}f\right)^{1/2}$ is the centralized root mean square (CRMS) bandwidth. When the center frequency is much larger than the CRMS bandwidth (a condition which is generally satisfied for radar) then $B_{RMS} \approx f_c$.

The above description corresponds the case of a *coherent* receiver. Such a receiver computes the crosscorrelation function of the pulse and the echo and estimates echo delay as the time corresponding to the maximum peak in the *fine structure* of the crosscorrelation function. An alternative type of receiver, the *semicoherent* receiver, estimates echo delay as the time corresponding to the maximum of the *envelope* of the crosscorrelation function between the pulse and the echo. For the semicoherent receiver, delay accuracy can be expressed by modifying the Woodward equation by substituting the signal CRMS bandwidth to the RMS bandwidth, so that $\sigma_s = (2\pi\ B_{CRMS}\ d)^{-1}$.

SNR breakpoint

Uncertainty in the delay estimate increases with noise. For relatively low levels of noise the time jitter falls within the central peak of the autocorrelation function and is inversely proportional to the SNR. However, when the noise level becomes comparable to the difference in amplitude between the center peak and the first side lobe, ambiguity in the choice of the correct peak arises. Figure 1 illustrates the effect of the noise level on the accuracy of the temporal measurement. Figure 1A shows a detail of the autocorrelation function in the neighborhood of the central peak for three pulses with the same bandwidth and different center frequencies (solid line: smallest f_c; dotted line: intermediate f_c; dashed line: largest f_c). The jitter in amplitude introduced by the noise is translated into a jitter in time that is controlled by the width of the central peak: the higher the center frequency, the smaller the jitter in time. However, when the noise level is of the order of the difference between the amplitude of the central peak and the first side lobe, the situation is reversed (Figure 1B). In fact, the difference in amplitude is smaller for higher center frequencies, so that signals with high center frequencies are more susceptible to peak ambiguity.

To study the effect of increasing levels of noise as a function of signal CRMS bandwidth and center frequency, we ran a set of Monte Carlo simulations. The pulses we considered are cosine packets of the form $\psi_{\sigma,\eta}(t) = K_{\sigma,\eta}\exp(-t^2/2\sigma^2)\cos(2\pi\eta t)$, where η is the center frequency, σ controls the spread in time of the pulse and its frequency bandwidth, and $K_{\sigma,\eta}$ is a normalization factor such that $\int_{-\infty}^{+\infty}\psi_{\sigma,\eta}^2(t)dt = 1$. This signal can be used without loss of generality. Analogous results would be obtained using different types of pulses with the same center frequencies and CRMS bandwidths used in our simulations. In each simulation white noise is added to the pulse to generate an echo, and the delay estimate is computed as the time corresponding to the maximum amplitude in the crosscorrelation between pulse and echo. In each set of simulations 200 realizations of the noise were generated. Different sets corresponded to pulses with different center frequencies and CRMS bandwidths. Figure 2a shows the root-mean-square error (RMSE) computed using the Monte Carlo simulations, for a fixed center frequency and CRMS bandwidth. Confidence intervals have been computed through bootstrapping, by sampling with replacement from the empirical distribution of the delay estimates obtained from the simulation. For high SNR's (region IV), performance is in accordance with the standard Woodward equation for the coherent receiver. As the SNR decreases, the performance shows a sharp transition (region III) to the modified version of the Woodward equation, corresponding to the semicoherent receiver (region II). For very low levels of SNR (region I), the intensity of the noise is so high that the accuracy rapidly decreases to zero. This behavior is common to all pulses. However the transition region is different according to the center frequency and CRMS bandwidth (figures 2b and 2c). Figure 2b shows the RMSE of the delay estimates as a function of signal to noise ratio in dB (SNR_{dB}) and CRMS bandwidth, for a fixed center frequency. For high signal to noise ratios, all signals follow the standard Woodward equation (Figure 2a). As the SNR decreases, signals with lower CRMS bandwidths are affected by peak ambiguity first, and their performance degrades to that of a semicoherent receiver. Signals with larger CRMS bandwidths are more resilient to peak ambiguity, and continue to perform according to the standard Woodward equation for even lower SNR's. The breaking point for each signal is marked with a square.

Figure 2c shows the case of a fixed CRMS bandwidth and different center frequencies. For high signal to noise ratios, all signals follow the standard Woodward equation (Figure 2a). As the noise level increases, signals with higher center frequencies are affected by peak ambiguity first, and their performance degrades to that of a semicoherent receiver. Signals with lower center frequencies are more resilient to peak ambiguity, and continue to perform according to the standard Woodward equation for lower signal to noise ratios.

Conclusion

We show that for increasing levels of noise the accuracy of the range estimate undergoes a sharp transition from the Woodward equation for a coherent receiver to a modified Woodward equation for a semicoherent receiver, due to ambiguity in the choice of the correct peak of the crosscorrelation between pulse and echo. We find that the breakpoint appears for lower SNR's in pulses with lower center frequencies and larger CRMS bandwidths, so that it is possible to optimize the pulse for a given SNR. The same ideas can be extended to the case of a fixed broadband signal, by performing the crosscorrelation at the receiver end separately in a set of frequency bands with the appropriate center frequencies and bandwidths. Auditory processing in biosonar involves transduction of wideband echoes in numerous narrow frequency bands, with independent gain control and thresholding at multiple levels in each band. This system encodes wideband signals in a time/frequency representation whose advantage may be to facilitate the adaptive shut-off of frequency channels where SNR is too low for coherent processing. The auditory brainstem processes the time-of-occurrence of spikes synchronized to sounds by successive stages of convergence and coincidence-detection to eliminate spikes that fall outside of the coincidence acceptance window. This neural mechanism might serve to prevent frequency channels from contributing to subsequent delay processing where spike timing is overly dispersed by noise.

Acknowledgments
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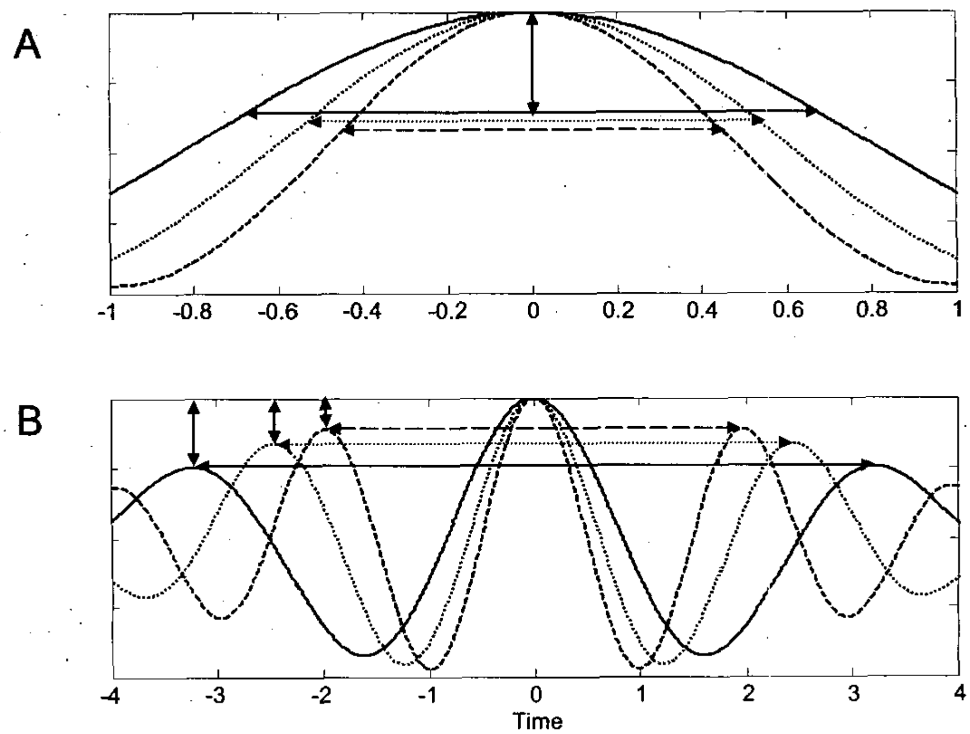

Figure 1. Effect of noise level on the accuracy of the temporal measurement. Plot A shows a detail of the autocorrelation function in the neighborhood of the central peak for three signals with the same bandwidth and different center frequencies (solid line: smallest f_c; dotted line: intermediate f_c; dashed line: largest f_c). The jitter in amplitude introduced by the noise is translated into a jitter in time that is controlled by the width of the central peak; the higher the center frequency, the smaller the jitter in time. However, when the noise level is of the order of the difference between the amplitude of the center peak and the first side lobe, the situation is reversed (Plot B). In fact, the difference in amplitude is smaller for higher center frequencies, so that signals with high center frequencies are more susceptible to peak ambiguity.

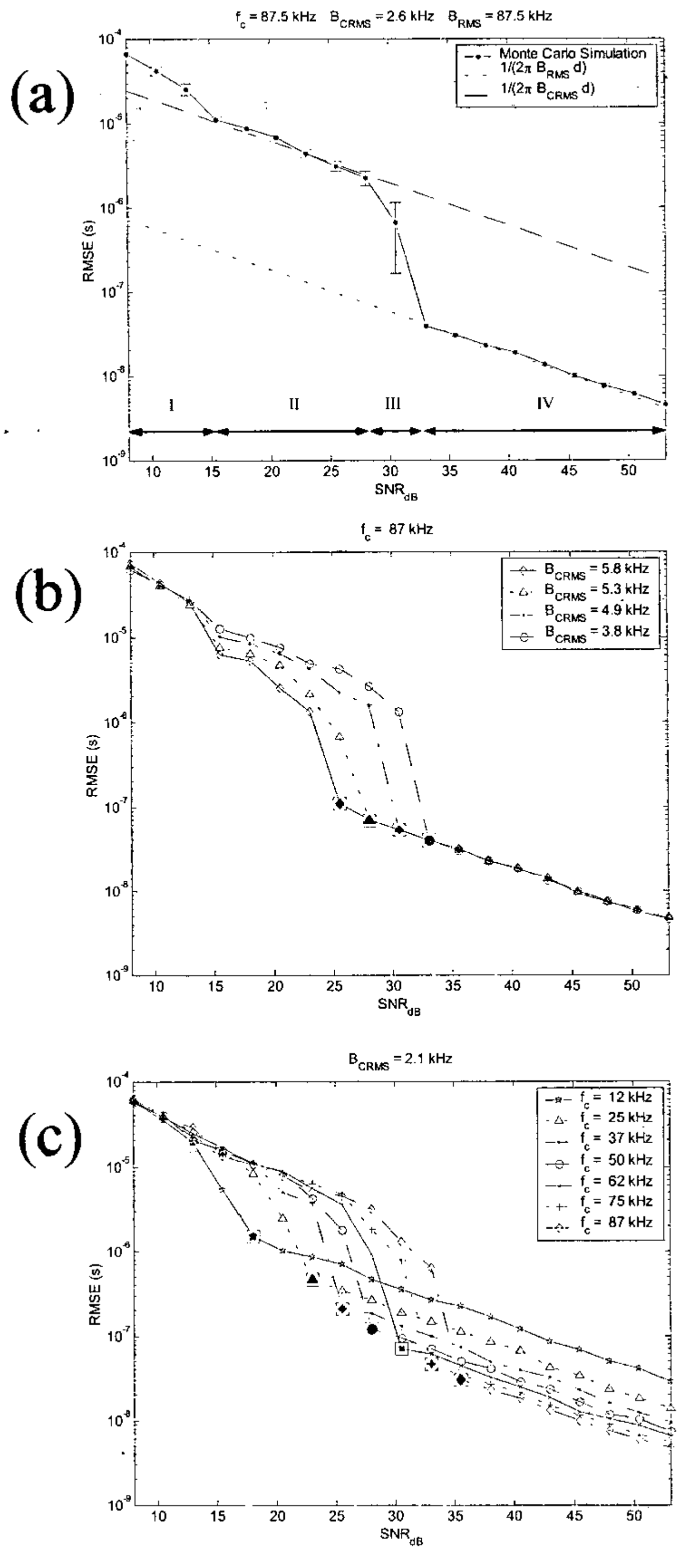
(a)
f_c = 87.5 kHz B_CRMS = 2.6 kHz B_RMS = 87.5 kHz
Monte Carlo Simulation
1/(2π B_RMS d)
1/(2π B_CRMS d)
RMSE (s)
SNR_dB
I
II
III
IV
(b)
f_c = 87 kHz
B_CRMS = 5.8 kHz
B_CRMS = 5.3 kHz
B_CRMS = 4.9 kHz
B_CRMS = 3.8 kHz
RMSE (s)
SNR_dB
(c)
B_CRMS = 2.1 kHz
f_c = 12 kHz
f_c = 25 kHz
f_c = 37 kHz
f_c = 50 kHz
f_c = 62 kHz
f_c = 75 kHz
f_c = 87 kHz
RMSE (s)
SNR_dB

Figure 2. Results of the Monte Carlo simulations. (a) RMSE of the delay estimates for a fixed center frequency and CRMS bandwidth; (b) RMSE as a function of signal to noise ratio in dB and CRMS bandwidth, for a fixed center frequency; (c) RMSE as a function of signal to noise ratio in dB and center frequency, for a fixed CRMS bandwidth.

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