# Time-frequency computational model for echo-delay resolution in sonar images of the big brown bat, *Eptesicus fuscus*

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## Abstract

To examine the basis for the fine (~2  $\mu$ s) echo-delay resolution of big brown bats (*Eptesicus fuscus*), we developed a time/frequency model of the bat's auditory system and computed its performance at resolving closely-spaced FM sonar echoes in the bat's 20-100 kHz band at different signal-to-noise ratios. The model uses parallel bandpass filters spaced over this band to generate envelopes that individually can have much lower bandwidth than the bat's ultrasonic sonar sounds and still achieve fine delay resolution. Because fine delay separations are inside the integration time of the model's filters (~250-300  $\mu$ s), resolving them means using interference patterns along the frequency dimension (spectral peaks and notches). The low bandwidth content of the filter outputs is suitable for relay of information to higher auditory areas that have intrinsically poor temporal response properties. If implemented in fully parallel analog-digital hardware, the model is computationally extremely efficient and would improve resolution in military and industrial sonar receivers.

## I. INTRODUCTION

The behavior of echolocating bats that emit frequency-modulated (FM) biosonar sounds shows that they create a detailed 3-dimensional representation of their immediate environment from processing echoes of these sounds (Neuweiler, 2000; Popper and Fay, 1995). The images these bats perceive incorporate the shapes of objects at their correct locations over the operating range of their sonar (e.g.,  $\sim 5$  m for big brown bats; Kick, 1982). Experimental evidence indicates that FM bats are capable of perceiving objects with a resolution of the order of millimeters and fractions of a millimeter (Simmons, et al., 1995; 1996, 1998). This performance is amazing because the neural representations that support the bat's images are formed by auditory midbrain and cortical neurons whose responses have a temporal precision of hundreds of microseconds to several milliseconds at best (Casseday and Covey, 1995; O'Neill, 1995; Dear, et al., 1993; Pollak and Casseday, 1989). Not only is the sharpness of the bat's images better than individual neurons seem able to sustain, but the computations required to place the spatial information the bats perceive into images from sonar signals are very intensive, involving large numbers of parallel temporal calculations. For example, the big brown bat (Eptesicus fuscus) transmits FM signals containing two to three harmonics that collectively span the band from 20 to 100 kHz. These broadcasts are beamed broadly, so they ensonify objects in nearly every direction, especially towards the bat's front. Consequently, echoes are effectively received from all the objects in the field of "view," which makes it necessary to explicitly form simultaneous distinct representations for each object so that they become segregated in perception (Simmons, et al., 1996). Then, the representation of each object derived from any single broadcast has to be integrated with the corresponding representations from previous and subsequent broadcasts so that the object's path can be tracked and its shape reconstructed as a single coherent picture from the succession of echoes (Grinnell, 1995; Kalko and Schnitzler, 1998) These types of calculations resemble computerized tomography or 3D reconstruction by rotating and overlaying individual images having less dimensionality, and they probably cannot be done at the early stages of the auditory pathways.

To create the bat's images, detailed information about the wideband, intrinsically high-resolution time-series FM waveforms of broadcasts and echoes first has to be converted into a representation capable of being transferred efficiently into higher auditory, presumably cortical areas, without using a bandwidth for transmission in any one neural channel that exceeds the surprisingly limited temporal response properties of the higher-level neurons that accept this information. Information necessary for measuring the arrival-times and arrival-time separations of multiple echoes has to survive the compression of the representation implied by the abrupt decrease in the temporal precision of neural responses between the auditory brainstem and the midbrain (Haplea, et al., 1994; Casseday and Covey, 1995) so the bat can perceive the objects with high resolution using its seemingly low-resolution computational elements. The desired representation presumably takes advantage of the ability of parallel neurons arranged in neuronal maps to carry the detailed information using multiple, parallel low-pass representations.

We propose a model of biosonar processing which performs a set of detailed measurements on echo returns in the brainstem and sends the outcome of these measurements using multiple low-bandwidth neuronal channels to higher areas of the auditory system, in the midbrain and cortex. This paper focuses on retention of high resolution for the delay of closely spaced echoes in a computational representation suitable for incorporating delay differences as range and cross-range information in spatial images of objects. In the context of delay resolution, we demonstrate the connection between the signal-to-noise ratio of the echoes and the resolution of echo separation, as would be embodied in neural responses in higher cortical areas. While it is not surprising that higher bandwidth is required for higher resolution, it is somewhat surprising that for a given echo resolution, lower signal-to-noise ratios require broaderrather than narrower-band processing. Normally systems are designed to focus on narrower signal bands when noise is strong to boost the energy available for overcoming the noise.

## **II. MODEL**

#### A. Background

Localization of objects from their echoes is a fundamental problem for analysis of acoustic signals. It is the basis of object exploration and scene analysis in sonar systems. Wideband active acoustic exploration of scenes relies on transmitting a series of pings, which impinge on objects and then return from different edges and surfaces. Distances to different parts of objects can be determined from the arrival-times of the individual replicas of the incident sound included in the overall return, and the cross-range locations of these parts can be determined from disparities in delay at two or more receiving points. The problem for understanding the performance of FM bats in such tasks as discrimination of airborne mealworms from disks (Griffin, et al., 1965) is how to conceive of the information necessary to reconstruct the range and cross-range appearance of objects being carried in the responses of neurons in the bat's auditory pathways.

When a sonar signal hits an object that is composed of several scattering points or planes (called "glints" in wideband parlance), there are multiple returns from the object. The delay between those returns gives an accurate indication of the structure in depth of object surfaces, or the separation of the glints in range. Using such delays at both ears, the bat may be able to achieve a complete reconstruction of the object in the rangecrossrange plane. The resolution of the sonar is determined by the smallest detectable temporal difference between echoes. Higher temporal resolution between echoes leads to a higher depth and shape resolution of objects. For high temporal resolution, the transmitted signal should be wideband. There also is a competing requirement for being able to detect targets at long range from weak echoes. The maximum range for object detection is governed by the energy of the signal—that is, by its amplitude and duration. Longer duration leads to higher integrated energy and thus longer operating range. Echolocating bats apply the classical "chirp-radar" solution to this problem—they transmit FM sounds whose bandwidth is kept high by the frequency span of the FM sweep, while energy is increased by increasing the signal's duration in conditions of low echo strength. However, this solution becomes complicated by the transducer design of mammalian hearing (Kössl and Vater, 1995; Ruggiero, 1992). The bat's inner ear receives the FM sweeps through parallel bandpass filters whose outputs are smoothed to

create an integration-time of several hundred microseconds for echo reception (Simmons, et al., 1995, 1996). The resulting auditory representation is a spectrogram-like timefrequency distribution for the energy in the sound made up of the envelopes of the bandpass-filtered, smoothed segments of the FM sweeps. The integration-time of the bat's auditory spectrograms is relatively long compared to the intrinsic time resolution of the bat's signals, which for big brown bats is crudely indexed to be about 12 µs from the reciprocal of the broadcast bandwidth (~ kHz). The much longer integration-time of the bat's auditory filters means that echoes arriving closer together in time than several hundred microseconds will merge together to form a single sound at separations far larger than the resolution limit of the signals themselves. Big brown bats nevertheless can assign two closely-spaced echoes their own arrival-times at far smaller separations. They readily distinguish echoes only 10-30 µs apart as separate, for example (Simmons, et al., 1995). The bat's limit of resolution at moderate signal-to-noise ratios is about 2 µs (Simmons, et al., 1998). When two echoes arrive closer together than the integrationtime, they mix to create an interference pattern of peaks and notches in the spectrum, where the frequency spacing of the notches is the reciprocal of the time separation of the echoes. If the big brown bat can resolve echoes as separate if they are only 10-30 us apart, it must use the interference pattern in *frequency* to estimate the separation in *time*. This conclusion implies that the bat uses its spectrogram-like time-frequency representation as more than just the format of its initial auditory code—it uses the properties of the two dimensions of time and frequency as the basis for computations to assemble spatial images of objects (Simmons, et al., 1996). Here, we examine the resolution attainable from a time-frequency representation of the big brown bat's FM signals for two echoes arriving at close time separations and different signal-to-noise ratios.

#### **B.** Outline of the model

Our model is based on the generation of multiple time/frequency templates representing ideal responses of the bat's auditory filters for different two-glint echo separations in the absence of noise. The model is evaluated by supplying input signals at different time separations and signal-to-noise ratios to test the ability of the timefrequency templates for resolving two-glint delay separations. The structure of the simulations resembles a psychophysical experiment on echo-delay resolution by bats. The filters that generate the frequency axis of the time/frequency representation are analogous to the cochlear block of the SCAT model (Saillant *et al.*, 1993). This model describes a comprehensive imaging system based on auditory-like time-frequency representations (Matsuo, et al., 2002; Peremans and Hallam, 1998; Saillant, et al., 1993). We modeled the action of the inner ear by a bank of band-pass filters followed by an envelope-smoothing process. The frequency selectivity of the basilar membrane in the bat's cochlea is simulated by 81 10<sup>th</sup>-order IIR Butterworth filters, with a constant Q=10, and center frequencies hyperbolically spaced between 25 kHz and 100 kHz. The excitation of the hair cells and primary auditory neurons is modeled by half-wave rectification and low-pass filtering using an IIR Butterworth 1<sup>st</sup>-order filter with a cut-off frequency of 3 kHz.

Figure 1 illustrates the effect of the different stages of the transduction filtering process on a frequency-modulated (FM) hyperbolic chirp with two harmonics, a start-

frequency of 100 kHz and an end-frequency of 25 kHz (Figure 1a). Figure 1b is the output of a band-pass filter with a center frequency of 40 kHz and Q=10. The output is then half-wave rectified (Figure 1c) and low-pass filtered (Figure 1d).

The effect of the entire filter bank is to encode the waveforms in the spectrogramlike time/frequency format characteristic of the mammalian auditory system. Figure 2 shows the output of the entire filter bank for three different signals. Each horizontal slice in the bottom plots corresponds to the output of one of the 81 cochlear filters after halfwave rectification and low-pass filtering. The signals in this example are based on a hyperbolic chirp with one harmonic. Since the center frequencies of the filters are hyperbolically spaced as well, the peaks in the response curves that trace the FM sweep fall on a straight line (the chirp's sweep is linearized).

If the echo is composed of distinct overlapping reflections from two closely-spaced glints, then interference occurs. In particular, if the delay between the two glints is larger than the integration time of the filters ( $\sim$ 300µs in our model), then the echoes from the two glints generate two separate response ridges in the time-frequency representation (Figure 2, right), and the distance from each glint to the emitted pulse can be computed independently entirely from the locations of the ridges in time. However, for smaller two-glint separations, it is no longer possible to distinguish between the two glints in the time domain because together they produce only one ridge. Interference gives rise to notches in the ridge whose number and location in frequency depend on the two-glint separation in time (Figure 2, center). Figure 2 shows these effects with a single harmonic to simplify the illustration, but the same effects occur for multiple harmonic signals. In our simulations we used a more biologically realistic chirp with two harmonics, the first harmonic spanning the interval between 25 kHz to 50 kHz. The difference between one and two harmonics covering the same 25-100 kHz band are shown in Figure 3.

In order to study the effects of background noise on the two-glint discrimination task, we generated band-limited white noise in the frequency range of the pulse -i.e. from 25 kHz to 100 kHz – and added this noise to the echoes used in the simulations. The signal-to-noise ratio (SNR) was computed according to Menne and Hackbarth, 1986:

$$SNR(dB) = 20\log_{10}\sqrt{\frac{2E}{N_0}}$$
(1)

where *E* is the total energy of the returning signal and  $N_{\theta}$  is the spectral density of the noise. Figure 4 shows the effect of different noise levels on the amplitude of a two-glint echo (A) and on its time frequency representation (B). Notice that, as the noise level increases (i.e. the SNR decreases), the location of the notches becomes increasingly more difficult to determine (Figure 4B, center). For very high noise levels the notches are almost completely masked by the noise (Figure 4B).

To build the model of echo-processing, a set of time-frequency templates was generated by applying the bandpass filter bank to a set of FM signals with different twoglint separations but no noise. These templates were then used as a set of parallel matched filters on the time-frequency representation of echoes mixed with noise created using the same bandpass filters. Figure 5 shows a collection of time-frequency templates used in the simulation. Each template corresponds to a specific two-glint separation, from 0  $\mu$ s to 48  $\mu$ s, with steps of 2  $\mu$ s. The resolution of this set of templates thus is 2  $\mu$ s. To explore higher resolution levels we generated a different set of templates in the same way, but using 0.2  $\mu$ s intervals from 0  $\mu$ s to 5  $\mu$ s in the two-glint separations. We will refer to the 2- $\mu$ s interval simulation as *coarse* and to the 0.2- $\mu$ s interval simulation as *fine*.

For a given two-glint separation  $\Delta t$  in the echo at a fixed SNR, we generated its time-frequency representation  $X(\Delta t)$  using the 81 cochlear filters described earlier. We then compared this representation to each time-frequency template Y(n) according to the following similarity measure:

$$M_{\Delta t}(n) = \frac{\sum_{ij} X_{ij}(\Delta t) \cdot Y_{ij}(n)}{\|X(\Delta t)\| \cdot \|Y(n)\|}$$

$$\tag{2}$$

The curve  $M_{\Delta t} = M_{\Delta t}(n)$  represents the responses of the collection of templates to the particular echo. The two-glint delay separation corresponding to the template with the highest response was used as our estimate  $\hat{\Delta}t$  for the two-glint separation  $\Delta t$ . This procedure was used for a series of two-glint echoes with different  $\Delta t$ 's. In particular, we  $\Delta t = 0 \mu s, 2 \mu s, 4 \mu s, \dots, 50 \mu s$  for the coarse resolution simulations and used  $\Delta t = 0 \mu s, 0.2 \mu s, 0.4 \mu s, \dots, 5 \mu s$  for the fine resolutions ones. For any given SNR, the family of response curves corresponding to different  $\Delta t$ 's can be combined together to form a surface, which we will refer to as the *response surface*. Since the delays in both the filters and the echoes are the same, i.e.  $\Delta t = 0 \mu s_1 2 \mu s_2 4 \mu s_1 \dots s_1 50 \mu s_2$  for the coarse resolution simulation, the correct estimates lay on the diagonal of the base plane of the surface. Figure 6 shows the response surface for single simulations of the coarse resolution experiment at three different noise levels. As the noise level increases (from top to bottom) the maximum response in some regions shifts away form this diagonal. For very low signal-to-noise levels the responses of each template to different two-glint separation in the echo are approximately equal (bottom figure), indicating a reduced discrimination power. Figure 7 shows the same surfaces for fine resolution simulations.

## III. RESULTS

Each of our simulated experiments with the model is performed with a specific SNR and two-glint separation  $\Delta t$  in the echo. The outcome of each experiment is the estimate  $\hat{\Delta}t$  of the two-glint separation. We performed a Monte Carlo simulation for each experiment, generating 20 realizations of the noise for each given SNR, and determining a new  $\hat{\Delta}t$  for each realization. The variability in the estimate of the two-glint separation can be visually assessed by looking at the histogram of  $\hat{\Delta}t$ . Each box in figure 8 and 9 shows the histogram of  $\hat{\Delta}t$  for a given Monte Carlo simulation at different two-glint separations (columns) and noise levels (rows). In each box a thin vertical line marks the position of the correct response. For large signal-to-noise ratios, the template with the highest response corresponds to the correct value of  $\Delta t$ , and all the points in the histogram fall in this correct bin. For lower SNRs, the model starts making mistakes, so that some

points in the histograms appear in the wrong bins (e.g. SRN=30, 25 in figure 8, SNR=45 in figure 9). For very low SNRs, most of the responses fall in the 0  $\mu$ s bin, for both the fine and the coarse resolution experiments. In addition, in the coarse experiment some of the responses cluster around the values 22  $\mu$ s and 46  $\mu$ s.

The bottom row of Figure 8 and 9 shows the combined histogram of the errors for all the Monte Carlo simulations with the same SNR, and provides a summary of the model's performance for a specific noise level. These summary histograms are centered on  $0\mu s$  as the nominally correct delay separation for all the different actual delay separations in the columns above. As the SNR decreases, the combined histogram becomes flatter and its spread around the 0  $\mu s$  center increases. Notice that for very low SNR there are only errors on the negative side of 0  $\mu s$ . This is a consequence of the bias towards the 0  $\mu s$  template for all the high noise levels mentioned earlier.

A major reason for doing these experiments was to evaluate the effects of changing a physiologically-relevant component of the model, the smoothing filter applied to the half-wave rectified output of each cochlear band-pass filter. Previous work has identified the frequency cutoff of this smoothing filter as the critical design parameter for echo transduction (Simmons, 1980). We compared the performance of various models that differ in the smoothing frequency (COF) of the low-pass filter in each of the 81 parallel cochlear band-pass channels. As expected, high COFs are more resilient to noise than low ones. Figure 10 and 11 show error histograms for different COFs (rows) and SNRs (columns). The band-pass filter case (BP only) corresponds to a COF=  $\infty$ . A summary plot for all the simulations discussed so far is shown in figure 12, which shows the root mean square error (RMSE) in µs versus signal-to-noise level in dB. Different curves correspond to different cut-off frequencies in the low pass filter; circles correspond to no halfway rectification and no low-pass filtering, i.e. the band-pass filters alone are used to create the time-frequency representation. The top figure shows the results for a collection of templates generated with increments of 2 µs in the two-glint separation. The dashed lines correspond to the case of 0.2-µs increments. Notice that for high SNRs, the coarse resolution lines all start at 2 µs, and the fine resolutions ones all start at 0.2 µs, since those are the maximal resolutions of the coarse and fine template sets. Form figure the fine resolution model is more sensitive to noise than the coarse one. In fact, the differences between the templates in the coarse resolution experiment are more substantial. The templates for the fine resolution experiments are more similar to each other, and lower levels of noise can easily mask those differences. In particular, the break points in SNR appear to be shifted between the two models by approximately 10 dB for high COFs, to 15-20 dB for lower ones.

### **IV. DISCUSSION**

The mammalian auditory system segments the frequencies in sounds into parallel frequency bands at the inner ear, smoothes the envelopes of the filtered signals, and transmits them in volleys of neural action potentials to higher brain centers for processing of acoustic information into auditory images. The inner ear of the little brown bat (*Myotis lucifugus*), a close relative of the big brown bat, contains about 900 inner hair cells (each comprising a single frequency-tuned channel), and the auditory nerve contains about 55,000 afferent fibers for transforming receptor activity into neural spikes

(Ramprashad, et al., 1978). These spikes are transmitted to a cascade of higher auditory centers (brainstem to midbrain to cortex) for processing the acoustic information they convey into images of objects. Nuclei at the first several stages of this cascade (brainstem) contain neurons numbered in thousands and tens of thousands with temporal and spectral response properties similar to the frequency tuning and time constants of the bandpass filters themselves. These neurons have to perform early auditory processing on the initial time-frequency representation so that it that can be efficiently relayed to higher auditory areas. In contrast, neurons in the inferior colliculus (midbrain) and auditory cortex are numbered in millions and have poor temporal response properties. Only their frequency tuning resembles that of the bandpass filters, and even this is modified by inhibitory responses at frequencies adjacent to excitatory frequencies (Casseday and Covey, 1995; Pollak and Casseday, 1989). These areas are candidates for performing the 3D scene analysis from acoustic data delivered in a time/frequency format from the brainstem. It seems likely that one function of the very large numbers of higher-level neurons is to support a representation of the spectrum of echoes with different degrees of frequency resolution to accommodate coarse to fine local spectral shape around each frequency. It is also likely that these auditory areas perform the binding of time/frequency information into perceived objects, so that the fine delay separations extracted from the interference pattern in frequency come to be associated with each target's correct absolute range extracted in the time-domain from the overall delay of echoes. Echoes that return from different objects at different distances and directions on successive pings have to be integrated into objects that are stable across multiple pings. The time scale for this kind of integration across pings necessarily is much longer than the 300 µs integration-time of the bandpass filters because it has to encompass the entire time interval from one ping to the next, which typically is 1 to 50 ms in bats. The temporal response properties of midbrain and cortical neurons are aligned with this longer epoch, implicating them in the interpretation and registration of stable acoustic The efficient relay of data from lower centers (brainstem) should convey scenes. information about the fine temporal structure of echoes while using low bandwidth channels to match the ascending information about each echo to the time scale of target scene analysis.

The echolocation of bats at ultrasonic frequencies requires perception of aspects of the detailed temporal structure of the waveform of echoes measured in microseconds, even though the neural channels for conveying this information upward to the auditory midbrain and cortex have much lower bandwidth and poor temporal response properties measured in milliseconds. Closely-spaced echoes fall inside the approximately 300 µs integration-time of the inner ear's bandpass filters, so their time separation is transposed into an interference pattern of amplitudes (notches and flanking peaks) at different Transmission of spectral shape in parallel frequency channels can be frequencies. achieved at low data rates compared to direct transmission of the equivalent time separations, and the auditory system may have evolved to exploit this feature of time/frequency coding so that higher auditory centers can have the long time constants required to assemble acoustic scenes across successive pings and still be able to receive and use information extracted from each echo. We have found that a time/frequency representation can be used to break the high-frequency, broad bandwidth signals received by the bat into parallel signals of greatly reduced frequency and bandwidth to register the interference pattern of overlapping echoes in a time-frequency representation so that channels with biologically realistic low temporal resolution can still account for the bat's fine delay resolution. We have specifically found that adequate resolution is achievable with parallel channels limited by input smoothing to frequencies no higher than 7 kHz. This representation is very efficient, being suitable for higher cortical areas to perform additional calculations on the fine temporal structure using slow neurons, and as such offers a valuable approach to high-resolution sonar receiver design.

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FIG. 1. (A) Hyperbolic chirp with two harmonics. (B) Output of a band-pass filter with center frequency of 40kHz and Q=10. (C) Output after halfway rectification. (D) Output of a low-pass frequency with COF=3kHz.



FIG. 2. FM hyperbolic echo with one harmonic and different two-glint separations (top row). Output of the filter-bank (bottom row). The band-pass filters have center frequencies hyperbolically spaced between 25 kHz and 100kHz and have constant Q=10. The low pass filter has a COF=3kHz.



FIG. 3. One harmonic versus two harmonics.



FIG. 4. (A) Echo with two glints separated by  $40\mu$ s embedded in band-limited noise for different signal-to-noise ratios. The frequency band of the noise is set to be equal to that of the pulse. (B) Time-frequency representation corresponding to the signals in (A).





to bottom) the maximum response moves away form the diagonal. For very low signalto-noise levels the responses of each template to different two-glint separation in the echo are approximately equal (bottom figure), indicating a reduced discrimination power. The above figures correspond to a COF = 7kH.





FIG. 8. Histogram of  $\Delta t$  estimates from the maximum responses of the templates for different two-glint separations (columns) and signal-to-noise ratios (rows). The bottom row shows the combined histogram of these estimates of delay separation for the entire collection of trials in the corresponding column (with  $\Delta t$  values centered on nominal values as zero). Results are given for a low-pass COF = 7kHz, and 2µs increments in the two-glint separations of the templates.







FIG. 11. Plots of summary histograms for errors in fine-resolution  $(0.2-\mu s \text{ steps})$  simulations across the entire collection of trials (bottom rows in Fig. 9) for different COFs and SNRs.



FIG. 12. Root mean square error (RMSE) in  $\mu$ s versus signal-to-noise level in dB. Different curves correspond to different cut-off frequencies in the low pass filter; circles correspond to no halfway rectification and no low-pass filtering, i.e. the band-pass filters alone are used to create the time-frequency representation. The solid lines in the top part of the graph shows the results for a collection of templates generated with coarse increments of  $2\mu$ s in the two-glint separation. The dashed lines in the lower part of the graph the case of fine 0.2 $\mu$ s increments.