# Graph Theory 0366-3267

Noga Alon, Michael Krivelevich

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## Homework Assignment No. 3

Due: Jan. 11, 2012

**1.** Let $G$ be a connected, simple graph with an even number of edges. Prove, using Tutte's Theorem, that the set of edges of $G$ can be partitioned into pairwise disjoint pairs, where each pair forms a path of length 2.

**2.** Let $A = (a_{i,j})$ be an $n$ by $n$ real matrix, where $n > 1$, $a_{i,j} \geq 0$ for all $i, j$ and the sum of elements in each row of $A$ and the sum of elements in each column of $A$ is exactly 1. Prove that there is a permutation $\sigma$ of $1, 2, \ldots, n$ so that $a_{i,\sigma(i)} > \frac{1}{n^2}$ for all $1 \leq i \leq n$.

**3.** Let $G$ be a simple 6-regular graph on 127 vertices. What is the chromatic index $\chi'(G)$ of $G$ ? Prove your claim.

**4.** Prove that the edges of every bipartite graph with minimum degree $\delta$ can be colored by $\delta$ colors so that every vertex is incident with an edge of every color. (Note: the required coloring is not necessarily a proper edge coloring.)

**5.** Prove that for every integer $k$ there is an integer $n = n(k)$ so that for any coloring of the set $Z_3^n$ of all $n$-dimensional vectors with coordinates in $Z_3$ by $k$ colors, there are three **distinct** vectors $x, y, z \in Z_3^n$ having the same color so that $x_i + y_i + z_i \equiv 0 \ (\ mod\ 3)$ for all $1 \leq i \leq n$.

**6.** (i) Is there a finite $n$ so that every simple, connected graph on at least $n$ vertices contains an **induced** subgraph with precisely 15 edges ? Prove or supply a counter-example.

(ii) Is there a finite $n$ so that every simple, connected graph on at least $n$ vertices contains an **induced** subgraph with precisely 19 edges ? Prove or supply a counter-example.