Graph Theory 0366-3267

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Homework Assignment No. 4

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1. Show that if

$$3\binom{n}{k}3^{-\binom{k}{2}} < 1$$

then there is a coloring of the edges of the complete graph on n vertices by 3 colors with no monochromatic clique of size k.

2. Let A be a set of $3m$ points in the Euclidean plane, and suppose that the distance between any two of these points is smaller than $\sqrt{2}$. Prove that the number of pairs P, Q of points of of A so that the distance between P and Q is at least 1 does not exceed $3m^2$.

3. Show that if the edges of a graph G can be covered by two trees then its chromatic number is at most 4.

4. Let G be a simple graph with maximum degree 7 containing no clique of size 4. Prove that the chromatic number of G is at most 6.

Hint: Show first that one can delete from G a bipartite graph leaving each degree in what's left at most 3.

5. Let G be a graph with chromatic number $\chi(G) = 11$ and with no cycle of length at most 20. Show that the number of vertices of G exceeds the population of China (which is less than 1,400,000,000).

6. Let G be a 2-connected, simple 5-regular planar graph drawn in the plane so that every face contains the same number of edges. What is the number of vertices of G ? Prove your claim.