NONREPETITIVE COLORINGS OF GRAPHS

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A sequence $S = s_1 s_2 ... s_{2n}$ is called a *repetition* if $s_i = s_{n+i}$ for each $i = 1, ..., n$. A coloring of the vertices of a graph G is *nonrepetitive* if no simple path of G looks like a repetition. The minimum number of colors needed for a nonrepetitive coloring of G is denoted by $\pi(G)$ and is called the *Thue chromatic number* of G.

The celebrated 1906 theorem of Thue [4] asserts that $\pi(P_n) = 3$ for all $n \geq 4$, where P_n is a path with n vertices. Let $\pi(d)$ denote the supremum of $\pi(G)$ where G ranges over all graphs with $\Delta(G) \leq d$. In [1] it was proved by the probabilistic method that there are absolute positive constants c_1 and c_2 such that

$$c_1 \frac{d^2}{\log d} \leq \pi(d) \leq c_2 d^2.$$

Recently Kündgen and Pelsmajer [2] proved that $\pi(G) \leq 4^t$ for graphs of treewidth at most t. Hence, by the result of Robertson and Seymour [3], any minor-closed class of graphs with unbounded Thue chromatic number must contain all planar graphs. This makes the following natural question even more intriguing:

Is the Thue chromatic number bounded for planar graphs?

A lot of similar variations involving graphs and combinatorics on words are possible. The talk will concentrate on several most interesting open problems of this type.

References

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