A sequence $S = s_1s_2...s_{2n}$ is called a repetition if $s_i = s_{n+i}$ for each $i = 1, ..., n$. A coloring of the vertices of a graph $G$ is nonrepetitive if no simple path of $G$ looks like a repetition. The minimum number of colors needed for a nonrepetitive coloring of $G$ is denoted by $\pi(G)$ and is called the Thue chromatic number of $G$.

The celebrated 1906 theorem of Thue [4] asserts that $\pi(P_n) = 3$ for all $n \geq 4$, where $P_n$ is a path with $n$ vertices. Let $\pi(d)$ denote the supremum of $\pi(G)$ where $G$ ranges over all graphs with $\Delta(G) \leq d$. In [1] it was proved by the probabilistic method that there are absolute positive constants $c_1$ and $c_2$ such that

$$c_1 \frac{d^2}{\log d} \leq \pi(d) \leq c_2d^2.$$ 

Recently Kündgen and Pelsmajer [2] proved that $\pi(G) \leq 4^t$ for graphs of treewidth at most $t$. Hence, by the result of Robertson and Seymour [3], any minor-closed class of graphs with unbounded Thue chromatic number must contain all planar graphs. This makes the following natural question even more intriguing:

Is the Thue chromatic number bounded for planar graphs?

A lot of similar variations involving graphs and combinatorics on words are possible. The talk will concentrate on several most interesting open problems of this type.

References


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