COMPLEX BRUNN-MINKOWSKI THEORY

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The classical Brunn-Minkowski theorem is an inequality for the volumes of convex combinations of convex bodies in $\mathbb{R}^n$. This talk is concerned with an analogous theorem for domains in $\mathbb{C}^n$ or complex manifolds, where convexity is interpreted as holomorphic convexity or pseudo-convexity - or in the case of compact manifolds as the Kähler condition. The main object of study is then not the volume of the domains, but L^2-norms of holomorphic functions (or sections of line bundles) on the domains. Even though this looks formally quite different from the classical real variable case, it turns out that the complex theory contains the real theory as the special case when we have enough (= toric) symmetry.

After introducing the set up and stating the main theorem I will briefly indicate applications to complex algebraic geometry, Kähler geometry and complex analysis. In the second lecture I will discuss some of the applications in more detail, try to give an idea of the proofs and explain more carefully the relations to classical Brunn-Minkowski theory.