Singularities of solutions of the Hamilton-Jacobi equation.
A toy model: distance to a closed subset.

by
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This is a joint work with Piermarco Cannarsa and Wei Cheng.

If $H : T^*M \to \mathbf{R}$ is a smooth Hamiltonian, and $f : M \times \{0\} \to \mathbf{R}$ is a smooth function, using for example generating functions (Chaperon & Viterbo), it is possible to extend f on to a Lipschitz, usually not C^1, function $F : M \times [0, +\infty[\to \mathbf{R}$ which is a (generalized) solution of the Hamilton-Jacobi equation

$$\partial_t F + H(x, \partial_x F) = 0.$$

In the case, where the Hamiltonian H is Tonelli, i.e. convex and superlinear in the momentum, we will give the local structure of the set $\mathrm{Sing}(F)$ of points where F is not differentiable. For example it is locally path-connected, and we will also study the homotopy type of $\mathrm{Sing}(F)$.

These studies do cover the case of singularities of the Euclidean distance function $d_A : \mathbf{R^k} \to [\mathbf{0}, +\infty[$ to a closed subset A of the Euclidean space $\mathbf{R^k}$. After stating the results in the general case, we will concentrate on the case of d_A to explain the methods of proof.