List of Theorems for the Exam

1) Every isometry f of $\mathbb{R}^n$ is of the form $f(x) = A(x) + b$, where $A \in O(n)$ and $b \in \mathbb{R}^n$.

2) The only finite plane symmetry groups are C_n and D_n (cyclic and dihedral groups).

3) Let $L \subset \mathbb{R}^2$ be a lattice, and let $S(L) = \{g \in \mathrm{Iso}^+(\mathbb{R}^2) \mid g(L) = L\}$. If $g \in S(L)$ is a rotation, then it must be of order 2, 3, 4 or 6 (crystallographic restriction).

4) In spherical geometry one has $\mathrm{Area}(\triangle ABC) = (\alpha + \beta + \gamma) - \pi$ (the radius of the sphere is assume to be 1).

5) Euler formula: for any convex polyhedron one has $V - E + F = 2$.

6) The spherical laws of cosines (I and II).

7) There is no local isometry between $\mathbb{R}^2$ and S^2.

8) The group $SO(3)$ is simple.

9) The isomorphism between $SO(3)$ and $\mathbb{R}P^3$ (using the quaternions).

10) Let $B \subset \overline{\mathbb{C}}$ be a disk. Then there is a unique circle in $\overline{\mathbb{C}}$ passing through two points $z_1, z_2 \in \mathrm{int}(B)$ and invaraint with respect to the reflection in ∂B.

11) The hyperbolic laws of cosines (I and II).

12) In hyperbolic geometry one has $\mathrm{Area}(\triangle ABC) = \pi - (\alpha + \beta + \gamma)$

13) Isometries of the hyperbolic plane: $\mathrm{Iso}^+(\mathbb{H}) = PSL(2, \mathbb{R})$ and $\mathrm{Iso}^+(\triangle) = PSU(1, 1)$.

14) The theorem about the angle of parallelsim (Bolyai-Lobachevsky theorem):

15) The fundamental polygon of the modular group.

16) Desargues' theorem.

17) Pappus's theorem.

18) The fundamental theorem of projective geometry (the case of $\mathbb{R}P^2$).