A Curious Example of Localization via Randomness in Classical Statistical Mechanics

In this talk we detail recent and ongoing work on randomness induced ordering is statistical mechanics.

The paradigmatic, but by no means only, example is given as follows: For each $i \in \mathbb{Z}^d$, let $\sigma_i \in \mathbb{S}^1$ be distributed according to Haar measure let $h_i = \alpha_i \hat{e}_2$ with the α_i i.i.d. ± 1 valued Bernoulli variables ($\hat{e}_2$ is the standard unit vector in $\mathbb{R}^2$ pointing vertically). Fixing a realization of the $\alpha_i's$, introduce the Hamiltonian

$$-\mathcal{H}(\sigma) = \sum_{\langle i,j \rangle} \sigma_i \cdot \sigma_j + \epsilon \sum_i h_i \cdot \sigma_i \tag{0.1}$$

a nearest neighbor spin system with corresponding Gibbs weight $e^{-\beta \mathcal{H}(\sigma)}$. The question we are interested in is whether, for any ϵ sufficiently small, there some β large enough for which sufficiently large block averages $\frac{1}{|\Lambda'|} \sum_{i \in \Lambda'} \sigma_i$ typically have preferred orientations under the Gibbs measure $Z^{-1} e^{-\beta \mathcal{H}(\sigma)}$. If so, what direction(s) are preferred?

In this talk we will explain why this is a remarkable question, both classically and quantum mechanically (the last if time permits). We then detail partial answers to the above questions.