BROWNIAN MOTION HOMEWORK ASSIGNMENT 4

INSTRUCTOR: RON PELED, TEL AVIV UNIVERSITY

(i) (a) Solve exercise 2.8 from the Brownian motion book.
(b) Let B be a standard one-dimensional Brownian motion. For $t \geqslant 0$, let

$$M(t) := \max_{s \in [0,t]} B(s).$$

Show that for each $t > 0$, the joint density of $(B(t), M(t))$ exists at each point (b, m), for $m \geqslant 0$ and $-\infty < b \leqslant m$, and equals

$$\frac{2(2m-b)}{\sqrt{2\pi t^3}} e^{-(2m-b)^2/2t}.$$

(ii) Read from the Brownian motion book the short section 2.2.3 (page 48) on the zero set of Brownian motion.
(a) Prove that any perfect set in $\mathbb{R}$ (a closed subset of $\mathbb{R}$ without isolated points) has the cardinality of the continuum. Thus, almost surely, the zero set of one-dimensional Brownian motion has the cardinality of the continuum.
(b) Solve exercise 2.6 from the Brownian motion book.

The Brownian motion book is available at: http://research.microsoft.com/en-us/um/people/peres/brbook.pdf

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