COMPLEXITY: Exercise No. 2
due next week

1. Prove that any 1-tape DTM with an infinite tape (i.e., the head of the machine can go any distance to the left or the right on the tape without limitations. Initially, the head is positioned on the first letter to the left of the input, and all the other letters on the tape are $\sqcup$) whose time complexity is $t(n)$ can be simulated by an ordinary 1-tape DTM with time complexity $t(n)$ (exactly!)

2. Show that a DTM with a two-dimensional "tape" (the tape is infinite in all four directions. Assume that in each step, the head can either stay in the same location or move one step left,right,up or down) whose time complexity is $t(n)$ can be simulated by a regular 3-tape DTM in $O(t(n)^2)$ time.

3. a. Write an efficient RAM program which receives a positive integer n and computes $\lfloor\sqrt{n}\rfloor$.
   b. Compute its time complexity (as a function of n).

4. Show that any RAM that runs in time $t(n)$ can be simulated by a RAM without indirect addressing in time $O(t^c(n))$ for some constant c. (assume that the input to the original RAM is encoded in a single input register).

5. a. Prove that if $t(n)$ is a complexity function such that $t(n_0) \leq n_0 + 1$ for some n_0 then any language in $\mathbf{TIME}(t(n))$ is regular (a language is regular if it can be accepted by a finite automaton).
   b. Show a language L in $\mathbf{TIME}(n+2)$ which is not regular.