COMPLEXITY: Exercise No. 3

due next week

1. Prove that for any $f(n)$ and $\epsilon > 0$, $\mathbf{SPACE}(f(n)) \subseteq \mathbf{SPACE}(\epsilon f(n))$.

2. Show that a language decided by a k-tape NDTM operating within time $f(n)$ can be decided by a 2-tape NDTM in time $f(n)$ (Assume that $f(n)$ is a complexity function which can be computate by a 2-tape DTM in time $O(f(n))$, and that $f(n) = \omega(n)$).

3. a. For a language L define $L^* = \{x_1 x_2 \cdots x_k : k \geq 0, x_1, \ldots, x_k \in L\}$. Show that if $L \in \mathbf{NP}$ then $L^* \in \mathbf{NP}$.
   b. Show that if $L \in \mathbf{P}$ then $L^* \in \mathbf{P}$.

4. a. Show that if f, g are proper complexity function then so are $f + g, f \cdot g, 2^g$ and $f(g)$ (for the last part, assume $f(n) \geq n$).
   b. Show that $\lceil \log n \rceil, n!$ and $\lceil n^{1/3} \rceil$ are proper complexity functions.

5. (Test 98)Prove or disprove: If f, g are proper functions, each can be computed in $O(2^n)$ time, then $h = f \cdot g$ (i.e., $h(x) = f(g(x))$) can be computed in $O(2^n)$ time.