COMPLEXITY: Exercise No. 5
due next week

1. For each of the following statements, prove, disprove or show that it is an open problem:

   (a) If $L \in \mathbf{NP}, L_1 \subsetneq L$ and $L_1 \in \mathbf{NP}$ then $L - L_1 \in \mathbf{coNP}$.

   (b) If $L \in \mathbf{NPC}, L_1 \subsetneq L$ and $L_1 \in \mathbf{SPACE}(\log n)$ then $L - L_1 \in \mathbf{NPC}$.

   (c) If $L \in \mathbf{NPC}$ and w is some fixed word then $\{xw : x \in L\} \in \mathbf{NPC}$. (Test 94)

2. Are the following problems **NP**-complete or polynomial? (prove)

   1/2 **SAT**:(Test 96)
   Instance: A CNF formula Φ.
   Question: Is there an assignment that satisfies Φ in which exactly half of the variables are TRUE?

   VERTEX COVER (VC):
   Instance: A graph $G = (V, E)$ and an integer k.
   Question: Does G have a vertex cover of size $\leq k$? (a vertex cover is a set $U \subseteq V$, such that for every edge $(u, v) \in E$ either $u \in U$ or $v \in U$ (or both))

   CLIQUE:
   Instance: A graph $G = (V, E)$ and an integer k.
   Question: Does G have a clique of size $\geq k$? (a clique is a set $U \subseteq V$, such that every two vertices from U are connected by an edge)

   DOMINATING SET (DS):(Test 92)
   Instance: A graph $G = (V, E)$ and an integer k.
   Question: Does G have a dominating set of size $\leq k$? (a dominating set is a set $U \subseteq V$, such that for every $v \in V \setminus U$ there is $u \in U$ such that $(u, v) \in E$)