COMPLEXITY: Exercise No. 7

due next week

1. Prove: If a problem is strongly **NP**-complete then the language obtained from it by representing all numbers in it in unary encoding is **NP**-complete.

2. Are the following problems **NP**-complete or polynomial? (prove)

   KERNEL:
   Instance: A directed graph $G = (V, E)$ (without loops).
   Question: Does G have a kernel ? (a kernel is a subset $V' \subseteq V$ such that no two vertices in V' are joined by an arc in E and such that for every vertex $v \in V - V'$ there is a vertex $u \in V'$ for which $(u, v) \in E$)

   FLOW WITH MULTIPLIERS: (Hint: use SUBSET SUM)
   Instance: A directed graph $G = (V, E)$, vertices $s, t \in V$, positive integer multiplier $h(v)$ for every vertex $v \in V - \{s, t\}$, positive integer capacity $c(e)$ for every $e \in E$, and a positive integer K.
   Question: Is there a *non-negative integer* function f such that:

   (a) $f(e) \leq c(e)$ for all $e \in E$.

   (b) for each $v \in V - \{s, t\}$, $\sum_{u:(u,v)\in E} h(v) f((u, v)) = \sum_{u:(v,u)\in E} f((v, u))$.

   (c) $\sum_{u:(u,t)\in E} f((u, t)) \geq K$.

3. Show that MAX CUT is **NP**-complete when the input graph is simple.