COMPLEXITY: Exercise No. 8

due next lesson

1. (Test 99)A graph $G = (V, E)$ has *bandwidth* k, if there is a bijection $f : V \to \{1, 2, \ldots, |V|\}$ such that $|f(i) - f(j)| \leq k$ for each edge $(i, j) \in E$.

   a. Give a polynomial time algorithm that given a graph G with bandwidth ≤ 5 and an appropriate mapping f, it finds a maximum independent set in G.

   b. Prove or disprove: The Independent Set problem is **NP**-complete on graphs with bandwidth $\leq \sqrt{|V|}$.

2. (Test 99)Is the following problem **NP**-complete? strongly **NP**-complete?

   Instance: Non-negative integers $a_1, a_2, \ldots, a_{2n}$.

   Question: Is there a set $S \subseteq \{1, 2, \ldots, 2n\}$ such that $|S| = n$ and $\sum_{i \in S} a_i = \sum_{i \notin S} a_i$?

3. (Test 98)Is the following problem **NP**-complete? strongly **NP**-complete?

   Instance: Positive integers $a_1, \ldots, a_n, b$.

   Question: are there non-negative integers $x_1, \ldots, x_n$ such that $\sum_{i=1}^{n} a_i x_i = b$.