COMPLEXITY: Exercise No. 9

due next lesson

1. (Test 98)Show that the following problem is in $\mathbf{NP} \cap \mathbf{coNP}$:
   Given an integer x (in binary encoding), does the number of prime divisors of x is a prime number?

2. Give an approximation algorithm for **MAXIMUM CLIQUE** whose ratio is $O(n/\log n)$ where n is the number of vertices in the input graph (i.e., there is a constant c such that $A(I) \geq \frac{c \log n}{n} OPT(I)$ for any instance I).

3. Consider the following greedy algorithm A for **MAX KNAPSACK**:

   (a) Reorder items so that $v_1/s_1 \geq v_2/s_2 \geq \cdots \geq v_n/s_n$.

   (b) for $i = 1$ to n do: insert item i into the knapsack if it still fits in the remaining space.

   (c) if the largest value of a single item is larger than the value of the solution obtained in (b), take that single item as the solution instead.

   Prove that $R_A = 2$.

4. Let Π be an integer-valued minimization problem and let Π_k be the decision problem of whether, for instance I, $\mathrm{OPT}(I) \leq k$. Prove that if Π_k is **NP**-complete for some constant k and $\mathbf{P} \neq \mathbf{NP}$, then a polynomial approximation algorithm A with $R_A < \frac{k+1}{k}$ cannot exist.