COMPLEXITY: Exercise No. 10
due next week

1. The class **FP** is the class of all binary relations R for which there is a polynomial DTM M_R such that for any input x, M_R outputs a string y such that $(x,y) \in R$, and if there is no such y then M_R outputs 'no'.

   The class **FNP** is the class of all binary relations R for which there is a polynomial NDTM M_R such that for any input x, there is a run of M_R in which the output is a string y such that $(x,y) \in R$ (if there is such y), and in all other runs of M_R, the output z satisfies either $(x,z) \in R$ or $z =$ 'no'.

   A function reduction from a binary relation R to binary relation R', is a pair (f,g) of functions that can be computed in polynomial times, such that for all x, y, $(f(x),y) \in R' \iff (x, g(y)) \in R$. If there is a function reduction from R to R' we denote $R \prec_f R'$

   (a) Show that **FP** and **FNP** are close under function reductions (i.e., if $R \prec_f R'$ and $R' \in$ **FP** then $R \in$ **FP** and the same for **FNP**).

   (b) Show that function reductions compose (i.e., if $R \prec_f R'$ and $R' \prec_f R''$ then $R \prec_f R''$).

   (c) Show that FSAT is **FNP**-complete.

2. (Test 99)Consider the following problem:
   Input: Sets $S_1, \ldots, S_k \subseteq \mathcal{N}$ where $|S_i| = 3$ for all i.
   Goal: Find a set $I \subseteq \{1, 2, \ldots, k\}$ with maximum size such that $S_i \cap S_j = \phi$ for all $i, j \in I$.
   Give a polynomial time approximation algorithm to this problem with a constant approximation ratio.

3. (Test 99)Consider the **MINIMUM STEINER TREE** problem:
   Input: A complete graph $G = (V, E)$, a subset of the vertices $X \subseteq V$, and a length function $l(e) > 0$ defined on the edges. The lengths satisfy the triangle inequality.
   Goal: Find a tree $T = (W, F)$ such that $X \subseteq W \subseteq V$, $F \subseteq E$ and Length$(T) = \sum_{e \in F} l(e)$ is minimal.
   Give an approximation algorithm for this problem whose approximation ratio is ≤ 2.
   Note: If $X = V$ then the optimal tree is a minimum spanning tree, but this is not true if $X \subset V$.

4. (Test 96)Define the **MULTIPROCESSOR SCHEDULING** problem:
   Input: positive integers $l_1, \ldots, l_n$ (lengths of tasks), and a positive integer m (the number of processors).
   Goal: Find a partition $S_1 \cup S_2 \cup \cdots \cup S_m = \{1, \ldots, n\}$ such that $\max_i \sum_{j \in S_i} l_j$ is minimum.
   Give a polynomial time approximation algorithm with a constant approximation ratio.